

# TSFS09 2A

November 3, 2025

## 1 Introduction

One main objective of this project is to give basic knowledge about turbocharging and how to model a turbo. Another is to show how, and why, downsizing and turbocharging can reduce fuel consumption and emissions compared to a naturally aspirated engine with equal performance. The project should also give insight into the main control loops for air control of a turbocharged engine.

The project goal is to model a downsized (1.2L) turbocharged petrol engine. A torque-based control structure with coordinated control of the actuators should then be developed for the engine model. The turbocharged engine's performance and emissions will be compared to the earlier projects naturally aspirated, and bigger (2.0L) petrol engine.

Here is a brief outline:

- **2A:** Collecting the models and the parameters they depend on.
- **2B:** Parametrizing the models with turbocharger map data.
- **2C:** Integrating the turbocharger with your engine model.

As you can see, very similar to project 1.

Project 2A is structured in two parts:

- A set of turbocharger questions to be answered.
- A set of models to collect and describe.

You will find the questions in the report template. Below you will find the suggested models.

## 2 Suggested Models for the Turbocharger

Turbochargers are simple in concept: harvesting the residual energy from the exhaust to drive a compressor makes the engine more powerful, i.e., a smaller engine feels larger. Combined with downsizing, a turbocharged engine may achieve the same performance with higher efficiency.

To fully represent a turbocharger mathematically, we need five component models. Mass flow and efficiency for the compressor and turbine, as well as a differential equation describing the rotational speed of the shaft that connects them.

### 2.1 Compressor Flow

Like the throttle and catalyst models, flow through the compressor ( $\dot{m}_c$ ) depends mainly on the pressure ratio  $\Pi_c$  (c for compressor) and rotational speed  $\omega_{tc}$  (tc for turbocharger):

$$\Pi_c = p_{02}/p_{01}, \quad U_2 = r_c \omega_{tc}, \quad \Pi_{c,\max} = \left( \frac{U_2^2 \Psi_{\max}}{2c_p T_{01}} + 1 \right)^{\frac{\gamma}{\gamma-1}}$$

$$\dot{m}_{c,\text{corr}} = \dot{m}_{c,\text{corr},\max} \sqrt{1 - \left( \frac{\Pi_c}{\Pi_{c,\max}} \right)^2}$$

Here,  $r_c$  is the impeller radius, which makes  $U_2$  the blade speed of the compressor. The unknown parameters are  $\Psi_{\max}$  and  $\dot{m}_{c,\text{corr},\max}$ .

### 2.2 Compressor Efficiency

The compressor efficiency ( $\eta_c$ ) model is posed in matrix form.

$$\eta_c = \eta_{c,\max} - \chi^T Q_\eta \chi, \quad Q_\eta = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix}$$

$$\chi = \begin{bmatrix} \dot{m}_{c,\text{corr}} - \dot{m}_{c,\text{corr}@\eta_{c,\max}} \\ \sqrt{\Pi_c - 1} - (\Pi_{c@}\eta_{c,\max} - 1) \end{bmatrix}$$

In total, six parameters needs to be estimated. They are  $\eta_{c,\max}$ ,  $\dot{m}_{c,\text{corr}@\eta_{c,\max}}$ ,  $\Pi_{c@}\eta_{c,\max}$ , and  $Q_{11}$ ,  $Q_{12}$ ,  $Q_{22}$ .

**Note:** Since the model for the efficiency is decreasing quadratically it will give unreasonably low or even negative efficiencies outside the normal operating region. You will have to put a lower saturation limit to the modelled efficiency, a suggestion is to use half the minimum value found in the map.

### 2.3 Turbine Flow

The suggested turbine flow ( $\dot{m}_t$ ) model is given as follows:

$$\text{TFP} = k_0 \sqrt{1 - \Pi_t^{k_1}}, \quad \Pi_t = \frac{p_{04}}{p_{03}} = \frac{p_{es}}{p_{em}}$$

The parameters to be estimated are  $k_0$  and  $k_1$ .

### 2.4 Turbine Efficiency

The suggested turbine efficiency ( $\eta_t$ ) model is given as follows:

$$\text{BSR} = \frac{\omega_{tc} r_t}{\sqrt{2c_p T_{03} \left(1 - \Pi_t^{\frac{\gamma-1}{\gamma}}\right)}}$$

$$\eta_t = \eta_{t,\max} \left(1 - \left[\frac{\text{BSR} - \text{BSR}_{\max}}{\text{BSR}_{\max}}\right]^2\right)$$

The parameters to be estimated are  $\eta_{t,\max}$  and  $\text{BSR}_{\max}$ .

**Note:** See note on compressor efficiency. Same applies here.

### 2.5 Turbocharger Shaft Dynamics

The turbine harvests energy from a hot pressurized exhaust stream, applying a positive torque  $M_t$  on the shaft. The compressor harvests energy to compress air, applying a negative torque  $M_c$  on the shaft. Additionally, there is friction in the turbocharger bearings. These three effects are balanced in the differential equation describing the turbocharger speed,

$$\dot{\omega}_{tc} = \frac{1}{J_{tc}} (M_t - M_c - M_{\text{fric}}), \quad M_{\text{fric}} = c_{\text{fric}} \omega_{tc}$$

where  $J_{tc} = 1 \times 10^{-5}$  is the rotational inertia of the shaft.  $c_{\text{fric}} = 1 \times 10^{-6}$  is a friction parameter.

### 2.6 Corrected Quantities

The following corrected quantities have been used for the turbocharger map:

$$\dot{m}_{c,\text{corr}} = \dot{m}_c \frac{\sqrt{T_{01}/T_{c,\text{ref}}}}{p_{01}/p_{c,\text{ref}}}, \quad N_{c,\text{corr}} = \frac{N_c}{\sqrt{T_{01}/T_{c,\text{ref}}}}$$

$$\text{TFP} = \text{TFP} \times \frac{\sqrt{T_{03} [\text{K}]}}{p_{03} [\text{kPa}]}, \quad \text{TSP} = \frac{N_t [\text{rpm}]}{\sqrt{T_{03} [\text{K}]}}$$