TSFS03 3rd Lab Tutorial

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Pontryagin's Maximum Principle (PMP)

Pontryagin's maximum principle is used in optimal control theory to find the best possible control for taking a dynamical system from one state to another, especially in the presence of constraints for the state or input controls. Consider the following dynamic system:

$$\dot{x} = f(x, u),\ x(0) = x_0,\ t \in [0, T] \tag{1}$$

where both x and u are a function of t, and the objective is to minimize J:

$$J = \int_0^T L(x, u)\ \mathrm{d}t \tag{2}$$

where Lagrangian L can be interpreted as the rate of cost for exerting control u in state x.Now consider construction of an ancillary function called Hamiltonian H using a arbitrary variable λ defined for all $t \in [0, T]$ by:

$$H(x, u, \lambda, t) = \lambda f(x, u) + L(x, u) \tag{3}$$

Pontryagin's maximum principle states that the optimal (achieving minimum J) state trajectory x^*, and optimal control u^* and corresponding multiplier λ^* must meet the following conditions:

$$\begin{aligned} &H(x^*, u^*, \lambda^*, t) \leq H(x, u, \lambda, t),\ \forall t \in [0, T] \\ &-\dot{\lambda} = \partial H / \partial x, \end{aligned} \tag{4}$$

At this point, the objective of minimizing (2) is changed to finding a variable λ^* which satisfies (4).

PMP in control of Hybrid Electric Vehicles

The nature of the state variables are obviously related to the dynamics of the system, which generally include mechanical, thermal, electrical, and electrochemical subsystems. Usually, for the purpose of energy management, HEVs can be described using quasistatic models. Thus the number of state variables strongly decreases, and the state vector can be reduced to include integral quantities such as the battery SoC, $x(t) \equiv SoC(t)$. One simple performance index for control of hybrid power train is the fuel mass m_s consumed over a mission of duration t_f. Hence J can be written as:

$$J = \int_0^{t_f} P_f(SoC(t), u(t))\ \mathrm{d}t \tag{5}$$

Where P_f is the fuel power at each time step. The Hamiltonian function can be shown in terms of power consumption:

$$H = \lambda P_e(SoC(t), u(t)) + P_f(SoC(t), u(t)) \tag{6}$$

That P_e is the batteries electrochemical power. The optimization problem can be summarized as finding the optimal SoC^*, u^*, and λ^* that satisfies the following conditions:

$$\begin{aligned} &H(SoC^*, u^*, \lambda^*, t) \leq H(SoC, u, \lambda, t),\ \forall t \in [0, T] \\ &-\dot{\lambda} = \partial H / \partial x, \end{aligned} \tag{7}$$