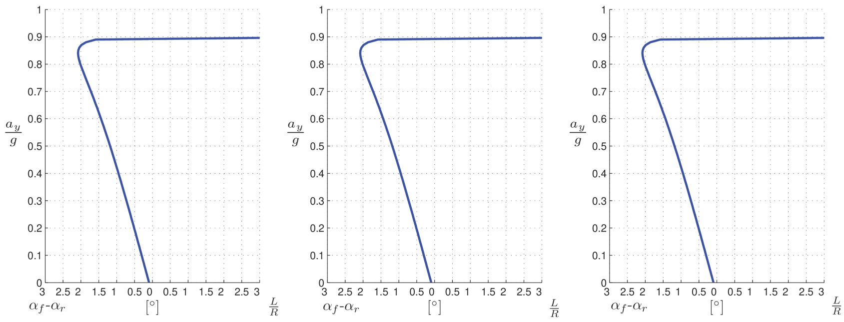
1. This is a continuation of exercise 5.1. The curves below show how $\alpha_f - \alpha_r$ depends on a_y/g. Draw curves that allows you to determine the following:

   a) The steer angle δ_f for constant curve radius $R = 100$ m.

   b) The steer angle δ_f for constant velocity 70 km/h.

   c) The quotient L/R for constant steer angle $\delta_f = 2.5°$.



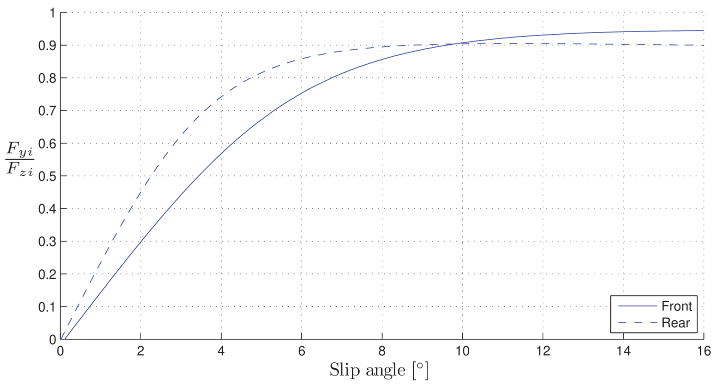
2. This is a continuation of exercise 5.1. Assume that $m = 1600$ kg, $I_z = 2800$ kgm^2, $l_1 = l_2 = 1.4$ m, and that the center of gravity is low enough so that longitudinal load transfer may be neglected. The car is traveling at 70 km/h and keeps a constant curve radius of 100 m when a braking force is applied on the rear wheels.

   a) Use the friction ellipse to determine F_{yr} if the braking force is $F_x = 0.5 \cdot F_{x,max}$.

   b) Determine the yaw acceleration $\dot{\Omega}_z$.

   c) How should δ_f change for $\dot{\Omega}_z = 0$ to hold instantaneously?

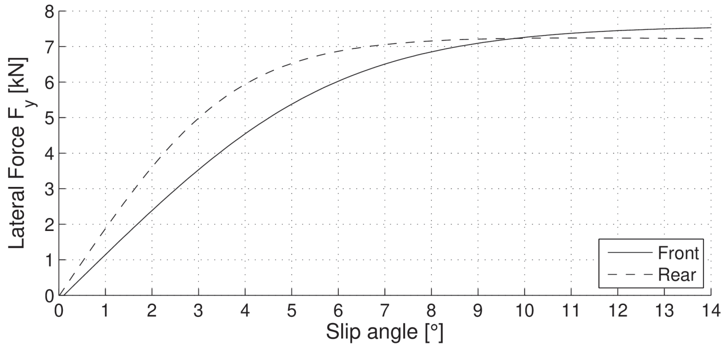
   The figure below is the tire-force characteristics from exercise 5.1.



3. In Lecture 4 the equation
$$\delta_f = \frac{L}{R} + K_{us}\frac{a_y}{g}$$
was derived for the steering angle at steady state cornering in the case of front-wheel steering. Assume that a rear-wheel steering is added to the model with the steer angle δ_r, as we did in Lecture 7. What is the corresponding relation?

4. It was shown in Lecture 7 that
$$\begin{bmatrix} V_y(s) \\ \Omega_z(s) \end{bmatrix} = (sM + A)^{-1}(\mathbf{u}_f\delta_f(s) + \mathbf{u}_r\delta_r(s))$$
$$= \frac{1}{\Delta}\begin{bmatrix} I_z s + a_4 & -a_2 \\ -a_3 & ms + a_1 \end{bmatrix}\left(\begin{bmatrix} 2C_{\alpha f} \\ 2l_1C_{\alpha f} \end{bmatrix}\delta_f + \begin{bmatrix} 2C_{\alpha r} \\ -2l_2C_{\alpha r} \end{bmatrix}\delta_r\right)$$
where
$$\Delta = I_z m s^2 + (I_z a_1 + m a_4)s + (a_1a_4 - a_2a_3).$$
Assume that $C_{\alpha f} = C_{\alpha r} = C_\alpha$ and $l_1 = l_2 = L/2$ and consider a step in the front steer angle, $\delta_f(s) = 1/s$. Use the initial value theorem, i.e., $f(0^+) = \lim_{s\to\infty} sF(s)$, to determine the immediate response in $\dot{V}_y$ and $\dot{\Omega}_z$ in the two cases $\delta_r = \delta_f$ (rear wheels in phase) and $\delta_r = -\delta_f$ (rear wheels out-of phase), respectively. (Hint: You don't need to calculate the matrix A.)

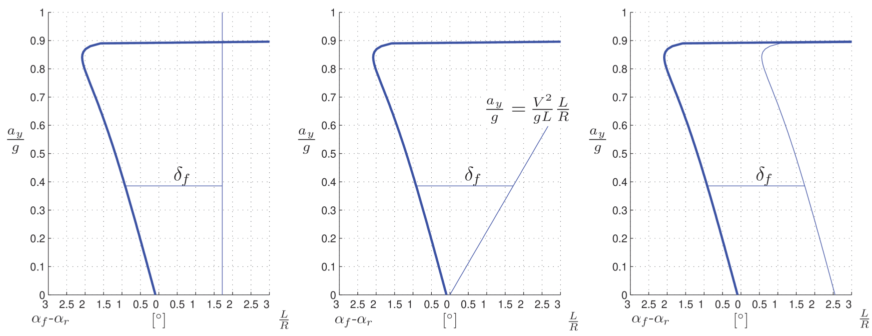
5. The lateral forces, as a function of the slip angle, for the front and rear tires respectively is given by the following figure:



The vehicle mass is $m = 1600$ kg, the wheelbase $L = 2.6$ m, and the center of gravity is 1.2 m behind the front axle. The vehicle is driving through a long curve with radius is $R = 100$ m (assume stationary conditions). Determine the steer angle δ_f if the velocity is $v = 90$ km/h.

Answers

1. Curves:



2. a) $F_{yr} = 2.6$ kN

   b) $\dot{\Omega}_z = 0.20$ rad/s^2

   c) δ_f should be reduced by about 0.2°.

3. $\delta_f - \delta_r = \frac{L}{R} + K_{us}\frac{a_y}{g}$

4. if $\delta_f = \delta_r$:

$$\begin{bmatrix} V_y(0^+) \\ \Omega_z(0^+) \end{bmatrix} = \begin{bmatrix} \frac{4C_\alpha}{m} \\ 0 \end{bmatrix}$$

   if $\delta_f = -\delta_r$:

$$\begin{bmatrix} V_y(0^+) \\ \Omega_z(0^+) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2LC_\alpha}{I_z} \end{bmatrix}$$

5. $\alpha_f \approx 5°$, $\alpha_r \approx 2.6°$, $\delta_f = 3.9°$