Vehicle Dynamics and Control

Lecture 8

The lectures

- Tyre modelling
- Longitudinal dynamics and control
- Lateral dynamics and control
- Vertical dynamics and control
- Stability and control
- Applications

Today's lecture

- Lateral dynamics: Stability of a car with a trailer
- Tyre modelling: Longitudinal and lateral forces
- Vertical dynamics: The quarter car model

Stability of a Car with a Trailer

Stability of a car with a trailer

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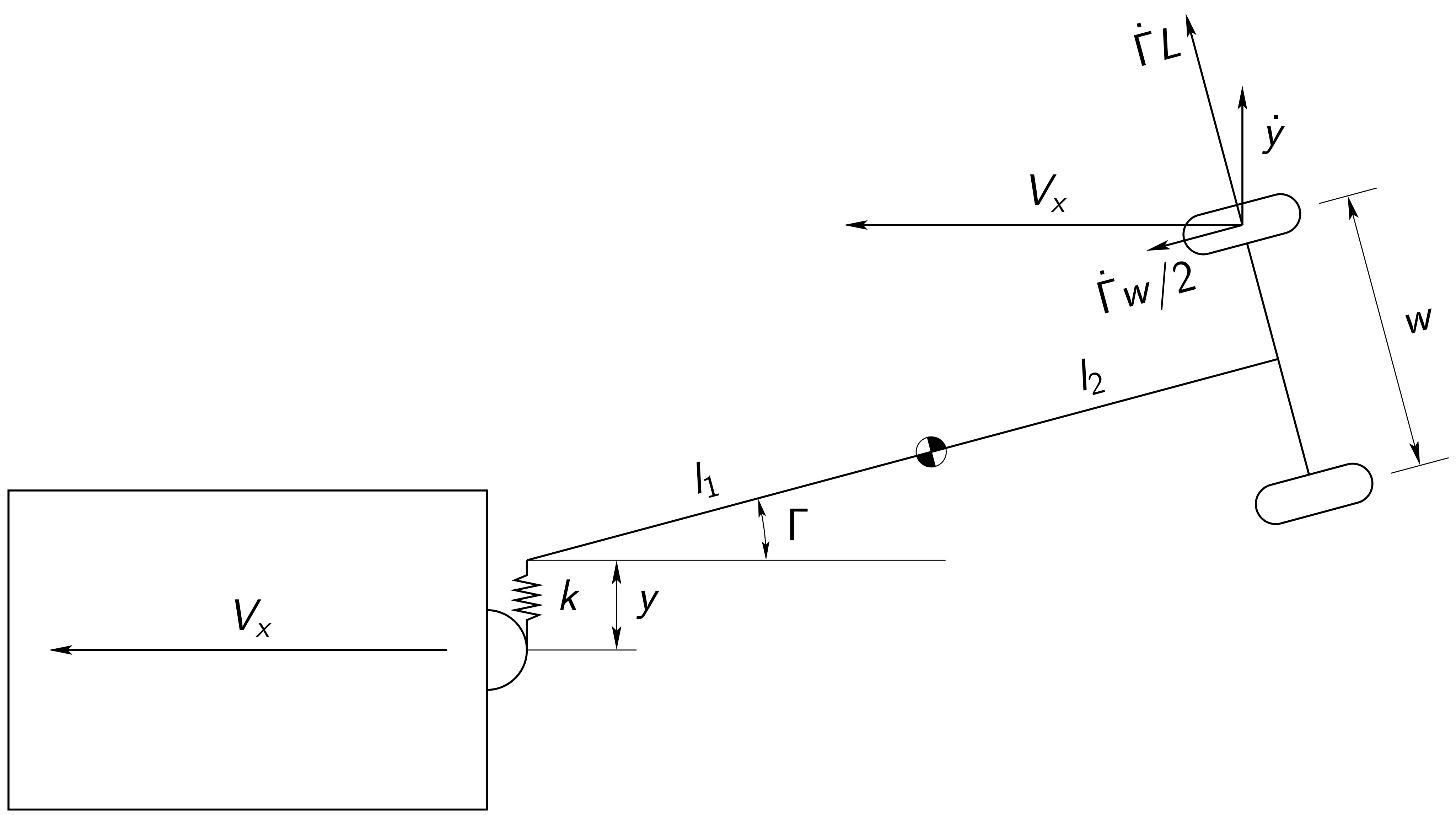
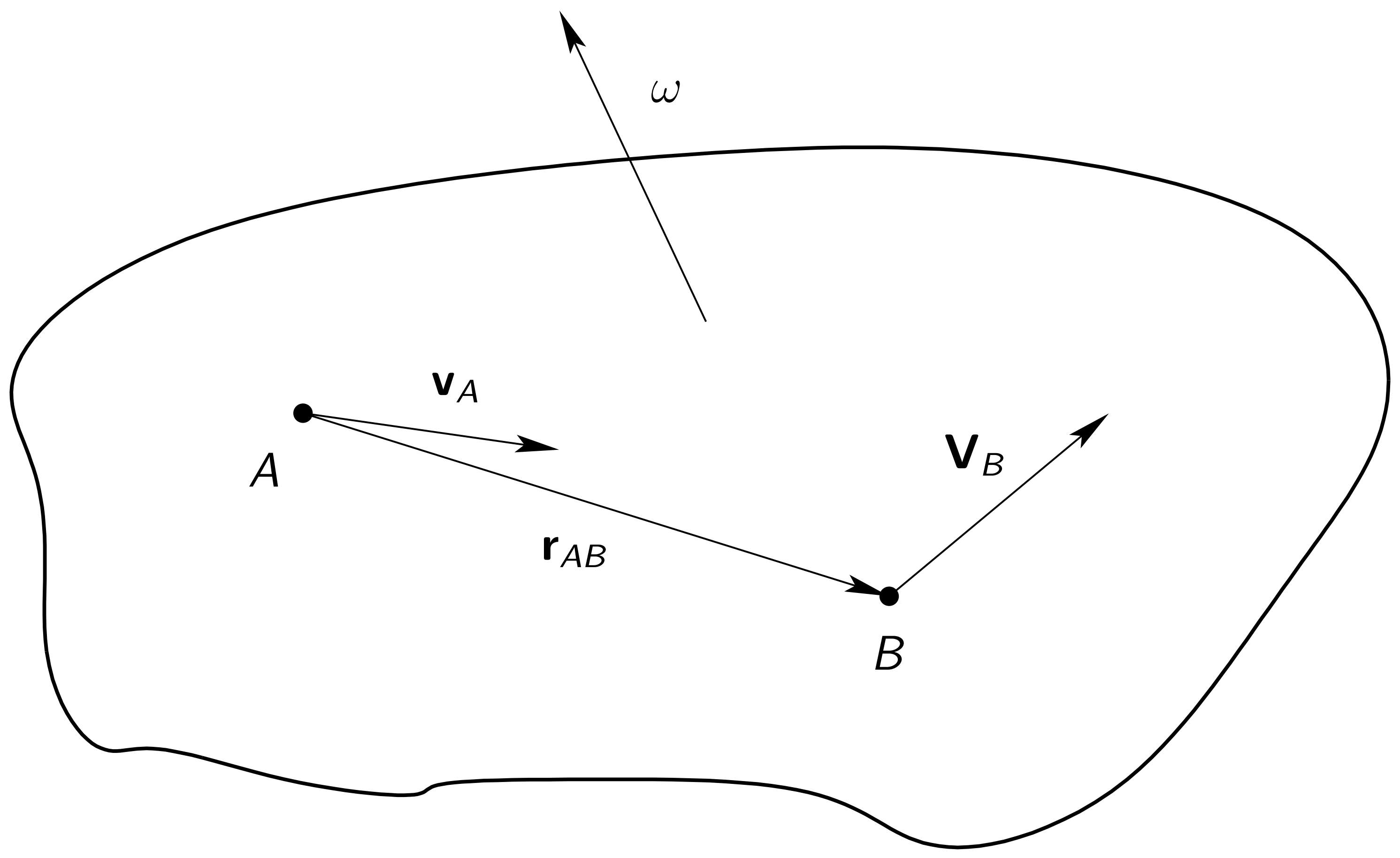
Car with a trailer: A model





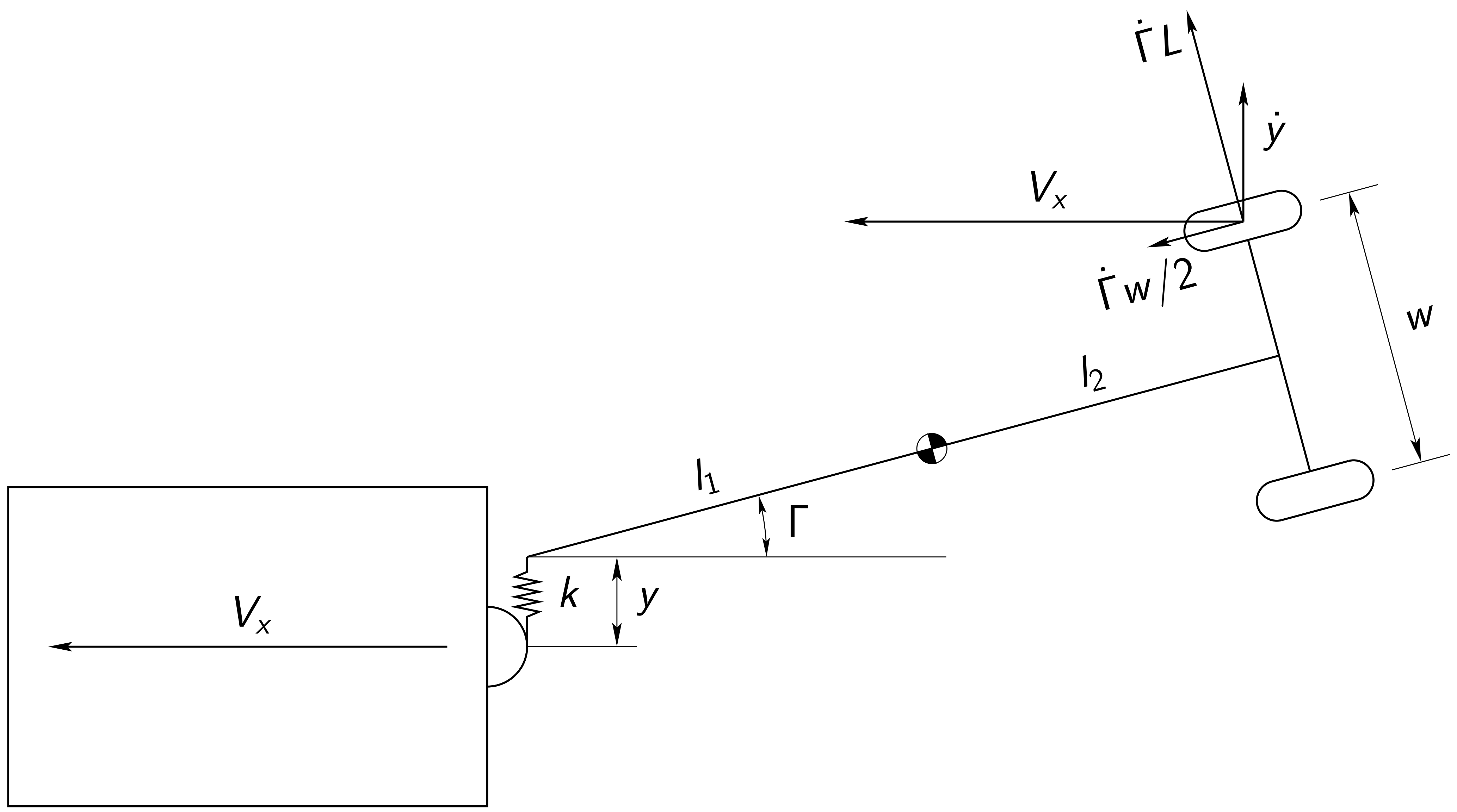
The formula $\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{AB}$, with A at the hitch and B at the right wheel, gives

$$\mathbf{v}_B = \begin{pmatrix} V_x \\ \dot{y} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\dot{\Gamma} \end{pmatrix} \times \left(L \begin{pmatrix} -\cos\Gamma \\ \sin\Gamma \\ 0 \end{pmatrix} + \frac{w}{2} \begin{pmatrix} \sin\Gamma \\ \cos\Gamma \\ 0 \end{pmatrix} \right)$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{AB}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{AB} + \omega \times (\omega \times \mathbf{r}_{AB})$$

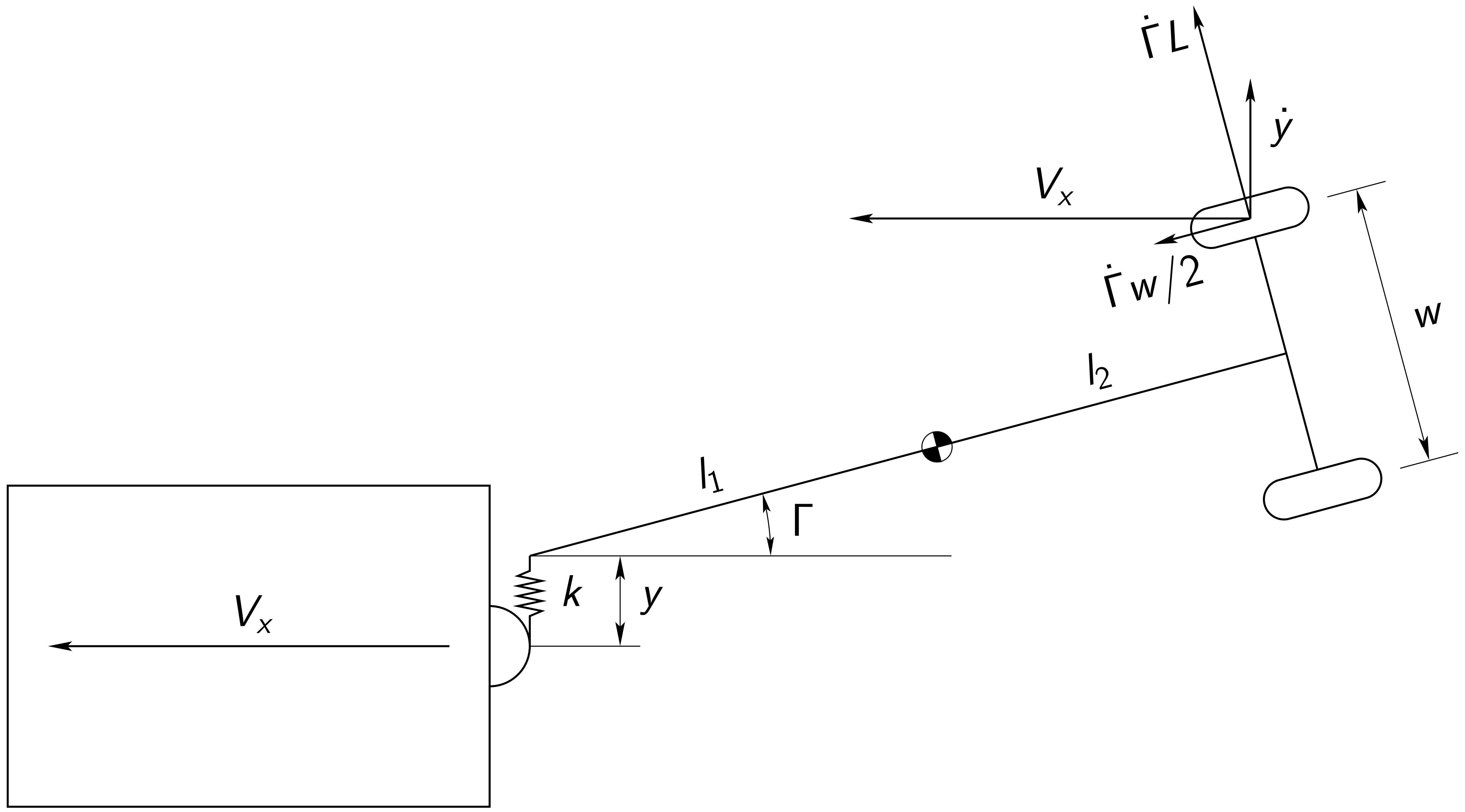
Car with a trailer: Kinematics



Calculate the cross-product:

$$\mathbf{v}_B = \begin{pmatrix} V_x \\ \dot{y} \\ 0 \end{pmatrix} + L\dot{\Gamma}\begin{pmatrix} \sin\Gamma \\ \cos\Gamma \\ 0 \end{pmatrix} + \frac{w\dot{\Gamma}}{2}\begin{pmatrix} \cos\Gamma \\ -\sin\Gamma \\ 0 \end{pmatrix}$$

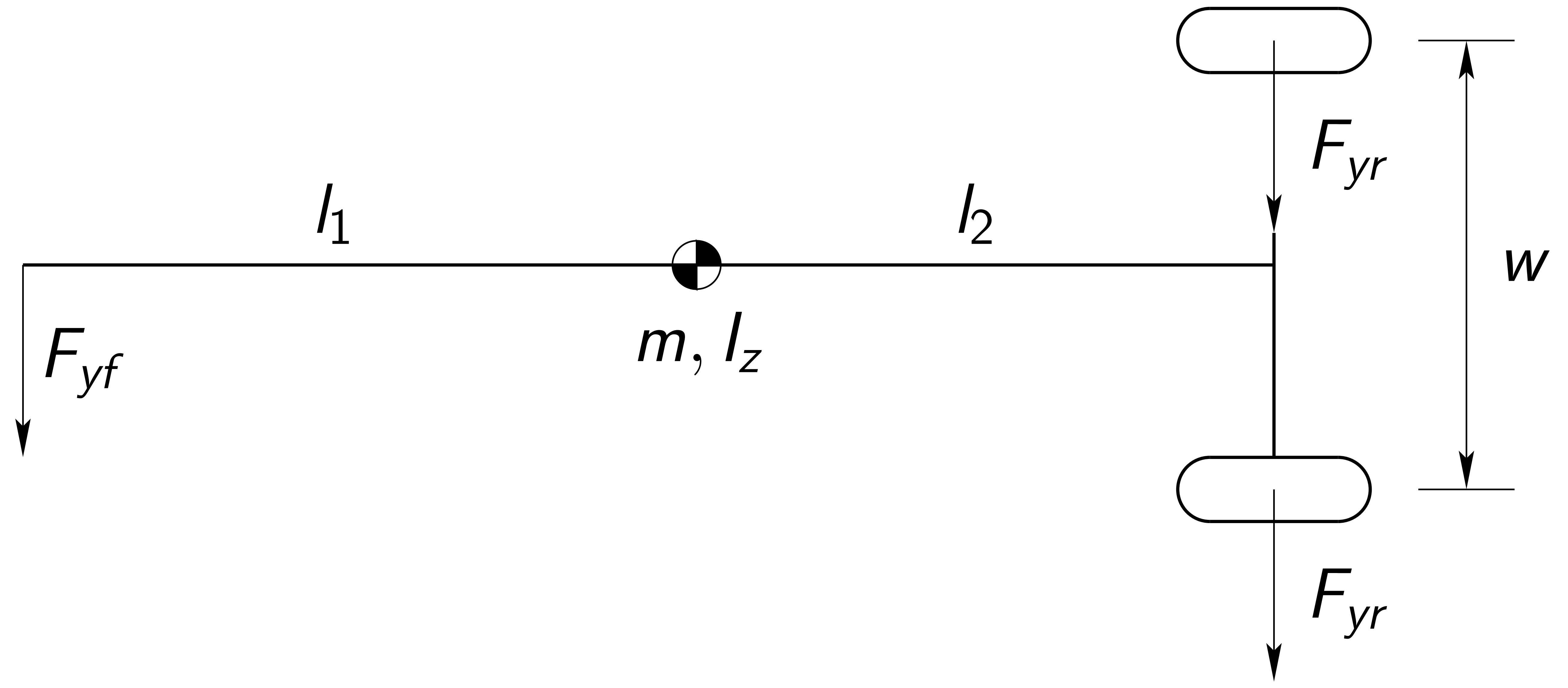
Car with a trailer: Kinematics



Approximation of the slip angle

$$\alpha \approx \frac{\Gamma V_x + \dot{\Gamma} L + \dot{y}}{V_x} = \Gamma + \frac{\dot{\Gamma} L}{V_x} + \frac{\dot{y}}{V_x}$$

Car with a trailer: Forces acting on the trailer



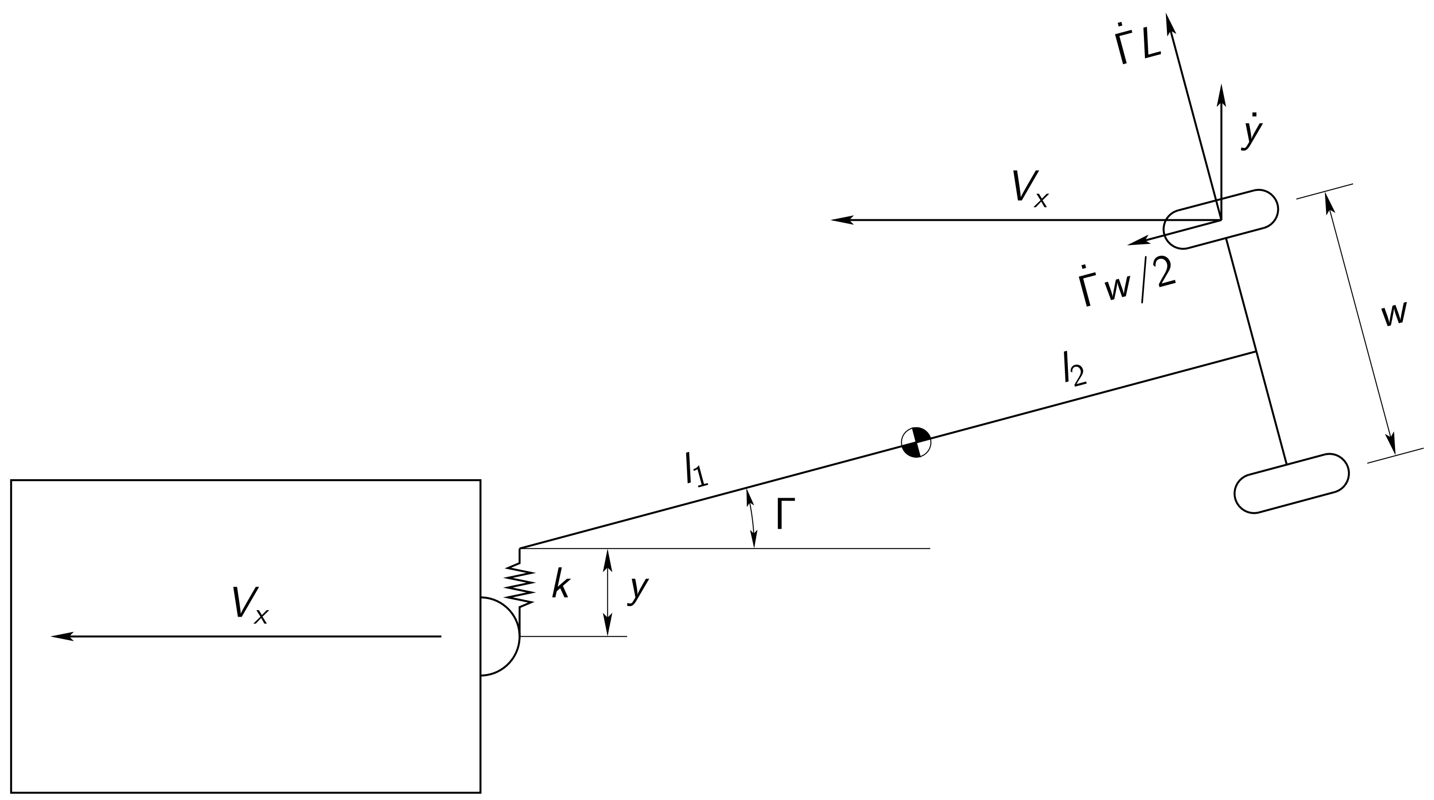
Model for spring stiffness:

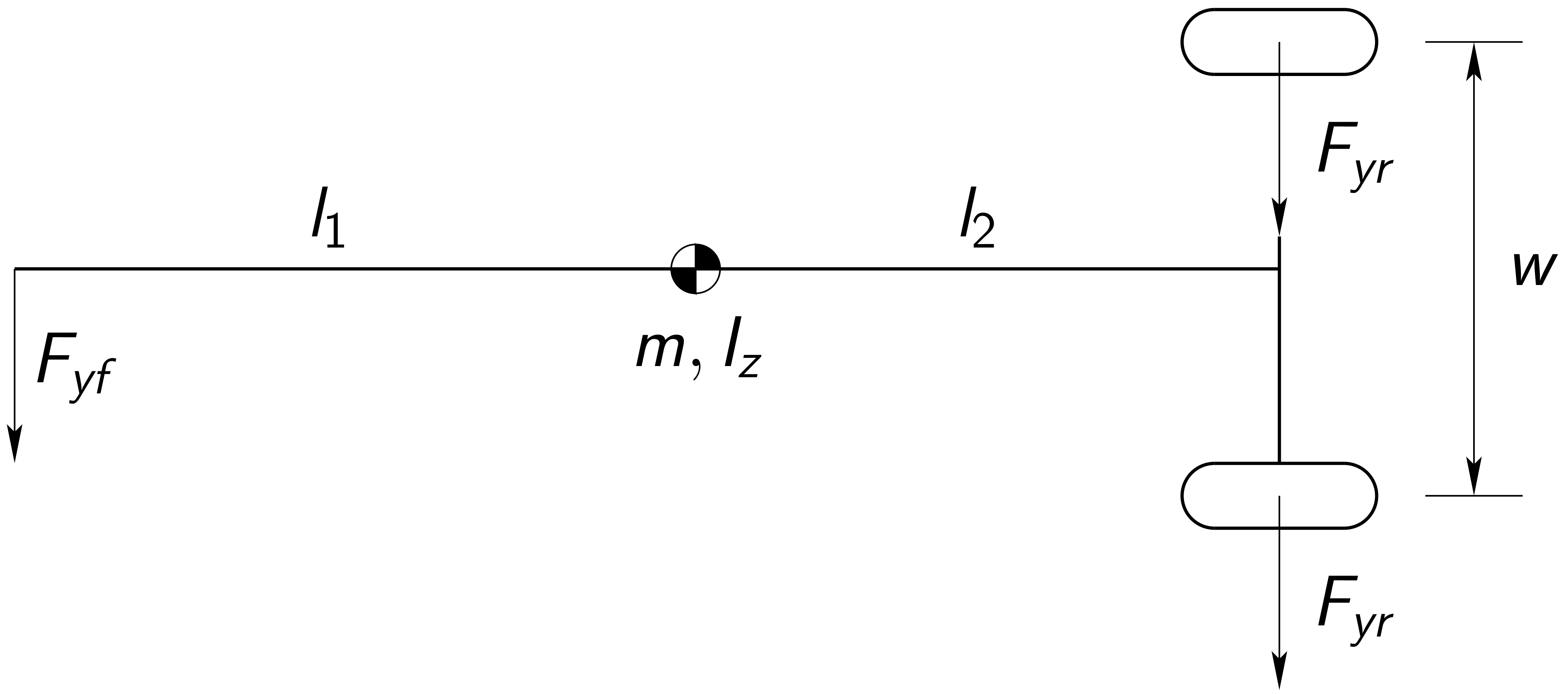
$$F_{yf} = ky$$

Model for tire force:

$$F_{yr} = C_\alpha \alpha = C_\alpha \left(\Gamma + \frac{\dot{\Gamma} L}{V_x} + \frac{\dot{y}}{V_x} \right)$$

Car with a trailer: Equations of motion





$$m\left(\ddot{y}+l_1\ddot{\Gamma}\right) = -F_{yf}-2F_{yr} = -ky-2C_\alpha\left(\Gamma+\frac{\dot{\Gamma}L}{V_x}+\frac{\dot{y}}{V_x}\right)$$

$$I_z\ddot{\Gamma} = l_1F_{yf}-2l_2F_{yr} = l_1ky-2l_2C_\alpha\left(\Gamma+\frac{\dot{\Gamma}L}{V_x}+\frac{\dot{y}}{V_x}\right)$$

Car with a trailer

From the previous slide:

$$m\left(\ddot{y}+l_1\ddot{\Gamma}\right)=-F_{yf}-2F_{yr}=-ky-2C_\alpha\left(\Gamma+\frac{\dot{\Gamma}L}{V_x}+\frac{\dot{y}}{V_x}\right)$$

$$I_z\ddot{\Gamma}=l_1F_{yf}-2l_2F_{yr}=l_1ky-2l_2C_\alpha\left(\Gamma+\frac{\dot{\Gamma}L}{V_x}+\frac{\dot{y}}{V_x}\right)$$

In matrix form

$$\begin{pmatrix} ml_1 & m \\ I_z & 0 \end{pmatrix}\begin{pmatrix} \ddot{\Gamma} \\ \ddot{y} \end{pmatrix}+\frac{1}{V_x}\begin{pmatrix} 2C_\alpha L & 2C_\alpha \\ 2l_2C_\alpha L & 2l_2C_\alpha \end{pmatrix}\begin{pmatrix} \dot{\Gamma} \\ \dot{y} \end{pmatrix}+\begin{pmatrix} 2C_\alpha & k \\ 2l_2C_\alpha & -l_1k \end{pmatrix}\begin{pmatrix} \Gamma \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Car with a trailer

In matrix form

$$\begin{pmatrix} ml_1 & m \\ I_z & 0 \end{pmatrix}\begin{pmatrix} \ddot{\Gamma} \\ \ddot{y} \end{pmatrix} + \frac{1}{V_x}\begin{pmatrix} 2C_\alpha L & 2C_\alpha \\ 2l_2C_\alpha L & 2l_2C_\alpha \end{pmatrix}\begin{pmatrix} \dot{\Gamma} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} 2C_\alpha & k \\ 2l_2C_\alpha & -l_1k \end{pmatrix}\begin{pmatrix} \Gamma \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Assume that the solution is in the form $\mathbf{x}(t) = e^{st}\mathbf{X}$ where s i a constant, possibly complex a number, and $\mathbf{X}$ is a constant vector. Substitute it into the system of differential equations:

$$e^{st}\begin{pmatrix} ml_1s^2 + (2C_\alpha L/V_x)s + 2C_\alpha & ms^2 + (2C_\alpha/V_x)s + k \\ I_zs^2 + (2l_2C_\alpha L/V_x)s + 2l_2C_\alpha & (2l_2C_\alpha/V_x)s - l_1k \end{pmatrix}\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

There exists a non-trivial solution $\mathbf{X}$ of the homogeneous system of equations if and only if s satisfies the characteristic equation:

$$\det\begin{pmatrix} ml_1s^2 + (2C_\alpha L/V_x)s + 2C_\alpha ms^2 + (2C_\alpha/V_x)s + k \\ I_zs^2 + (2l_2C_\alpha L/V_x)s + 2l_2C_\alpha(2l_2C_\alpha/V_x)s - l_1k \end{pmatrix}$$

$$= mI_zs^4 + \frac{2C_\alpha}{V_x}(I_z + ml_2^2)s^3 + (k(I_z + l_1^2m) + ml_2C_\alpha)s^2 + \frac{2C_\alpha}{V_x}kL^2 + 2C_\alpha Lk = 0$$

Car with a trailer: Routh's stability criterion

Consider a third order equation:

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$$

The Routh array is defined as:

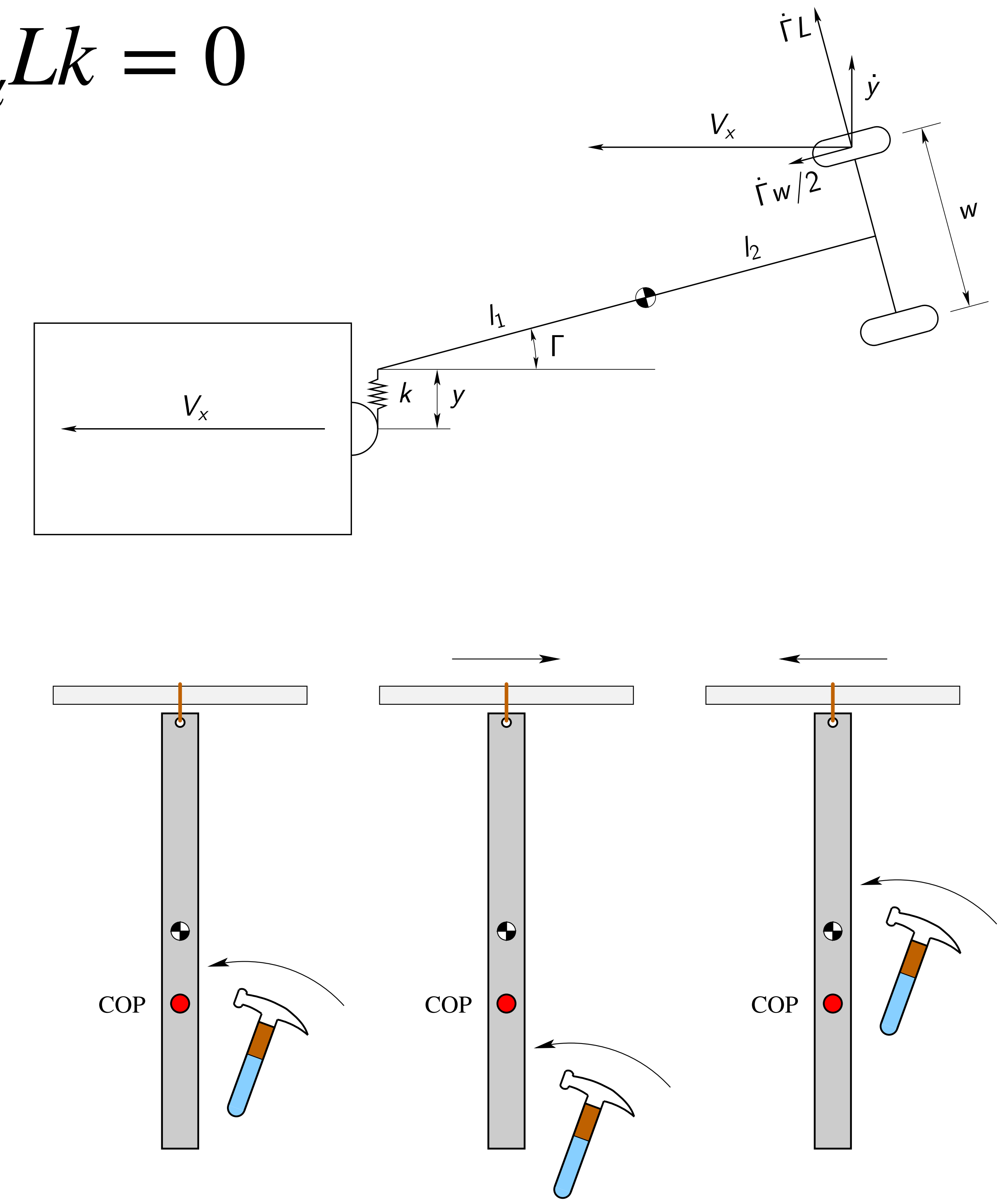
$$\begin{array}{cc} a_0 & a_2 \\ a_1 & a_3 \\ \frac{a_1 a_2 - a_0 a_3}{a_1} & 0 \\ a_3 & 0 \end{array}$$

Routh's stability criterion: All solutions in the left half-plane if and only if $a_0, a_1, a_3 > 0$, and $a_1 a_2 - a_0 a_3 > 0$.

Car with a trailer: A third-order model, stability

We are studying the determinant:

$$mI_z s^4 + \frac{2C_\alpha}{V_x}(I_z + ml_2^2)s^3 + (k(I_z + l_1^2 m) + ml_2 C_\alpha)s^2 + \frac{2C_\alpha}{V_x}kL^2 s + 2C_\alpha Lk = 0$$



Assume that C_α is large, then we get

$$\frac{2(I_z + ml_2^2)}{V_x}s^3 + ml_2 s^2 + \frac{2kL^2}{V_x}s + 2Lk = 0$$

The Routh's stability criterion gives

$$\frac{2kL}{V_x}(ml_2 L - (I_z + ml_2^2)) > 0$$

and

$$ml_1 l_2 > I_z$$

i.e. the center of percussion should be located in front of the axle of the trailer.

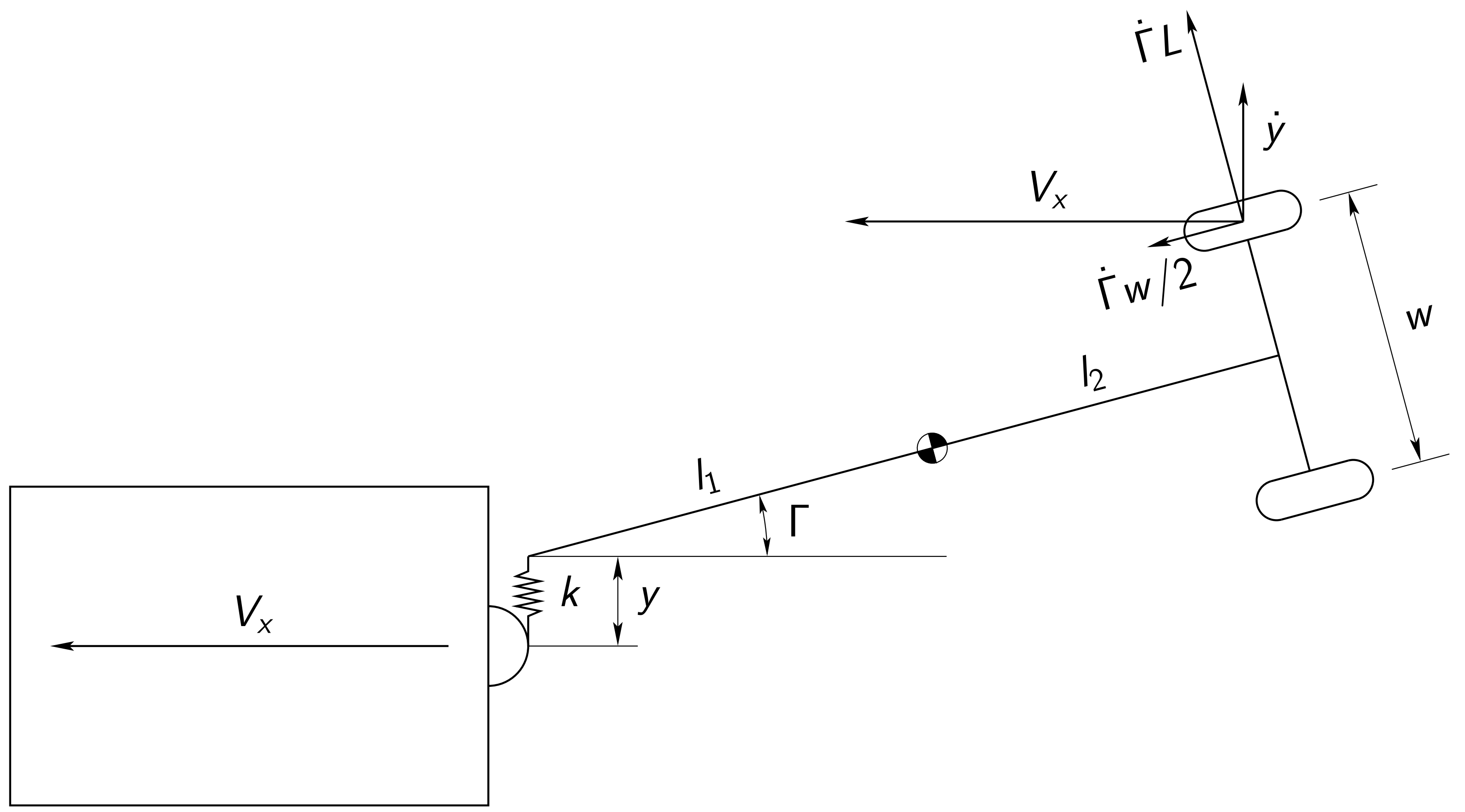
Car with a trailer: A third-order model, stability

Add damping at the hitch $M = C\dot{\Gamma}$ where C is an effective rotary damping constant.

$$\frac{2(I_z + ml_2^2)}{V_x}s^3 + (C/V_x + ml_2)s^2 + \frac{2kL^2}{V_x}s + 2Lk = 0$$

The Routh's stability criterion then becomes

$$\frac{2kL}{V_x}\left(\frac{CL}{V_x} + ml_2L - (I_z + ml_2^2)\right) > 0$$



$$\frac{CL}{V_x} + ml_1l_2 > I_z$$

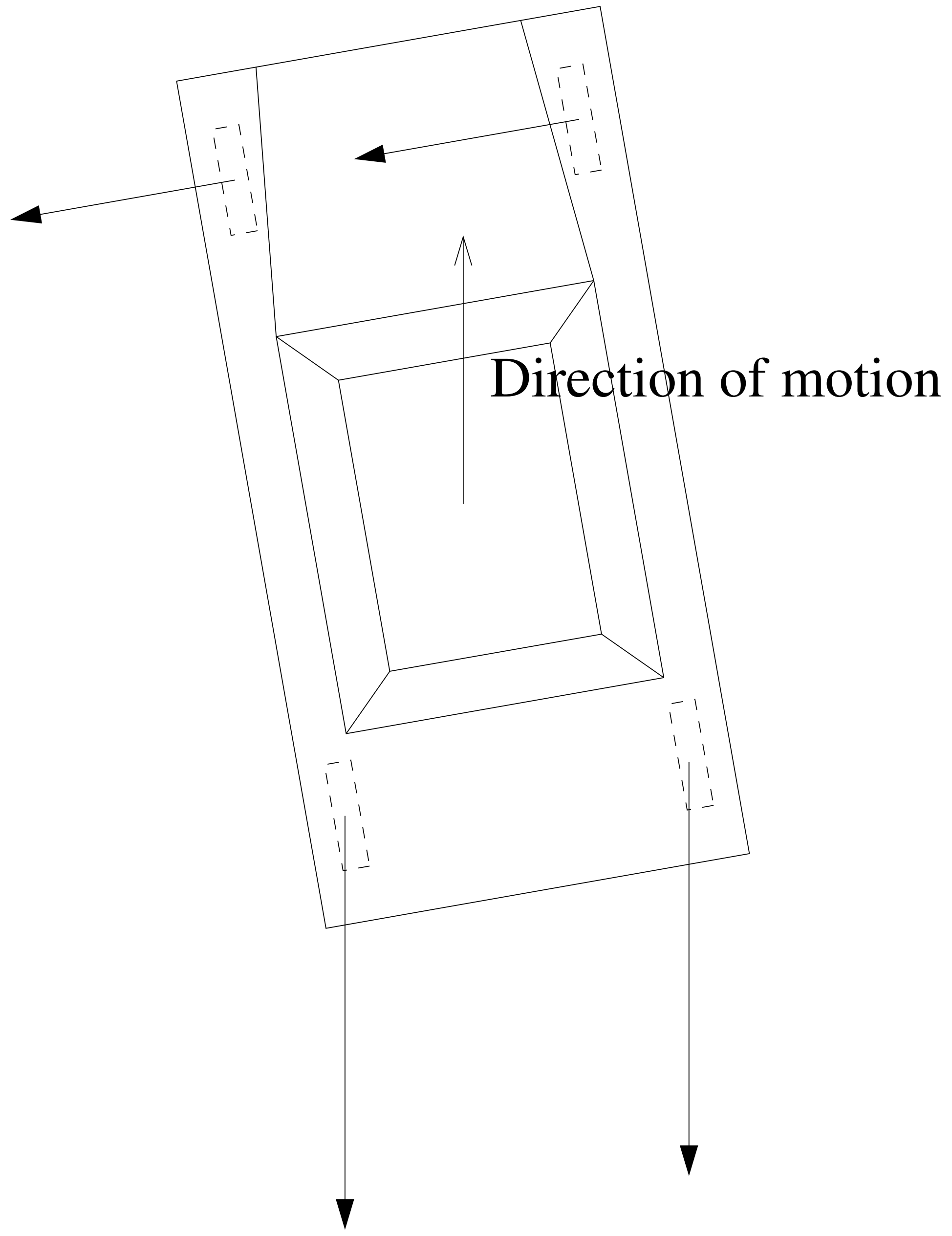
It is not fulfilled if $ml_1l_2 < I_z$ and

$$V_x \geq \frac{CL}{I_z - ml_1l_2}$$

Tyre Modelling: Lateral and Longitudinal Forces

Lateral Forces and Stability: Braking

Assume that the rear wheels are used for braking causing a wheel lock-up:



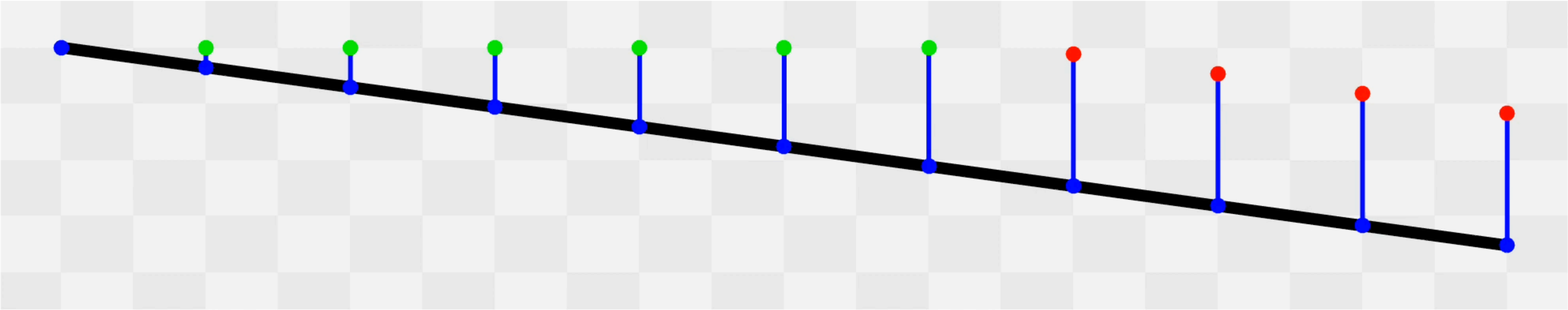
The figure shows that the car will turn away from the intended direction and the car will probably become unstable.

Från föreläsning 2
Idag: Lägger på longitudinella krafter

Borstmodellen

Borstmodellen kan användas även för att bestämma laterala krafter.

Stråna böjs nu ut i sidled. Sett uppifrån:



De gröna cirklarna representerar punkter där den nedre delen av strået har kontakt med underlaget utan att glida och de röda där strået har börjat glida.

Laterala och longitudinella krafter

Vi har tidigare studerat laterala och longitudinella krafter separat
Fallet med både laterala och longitudinella krafter är mer komplext.

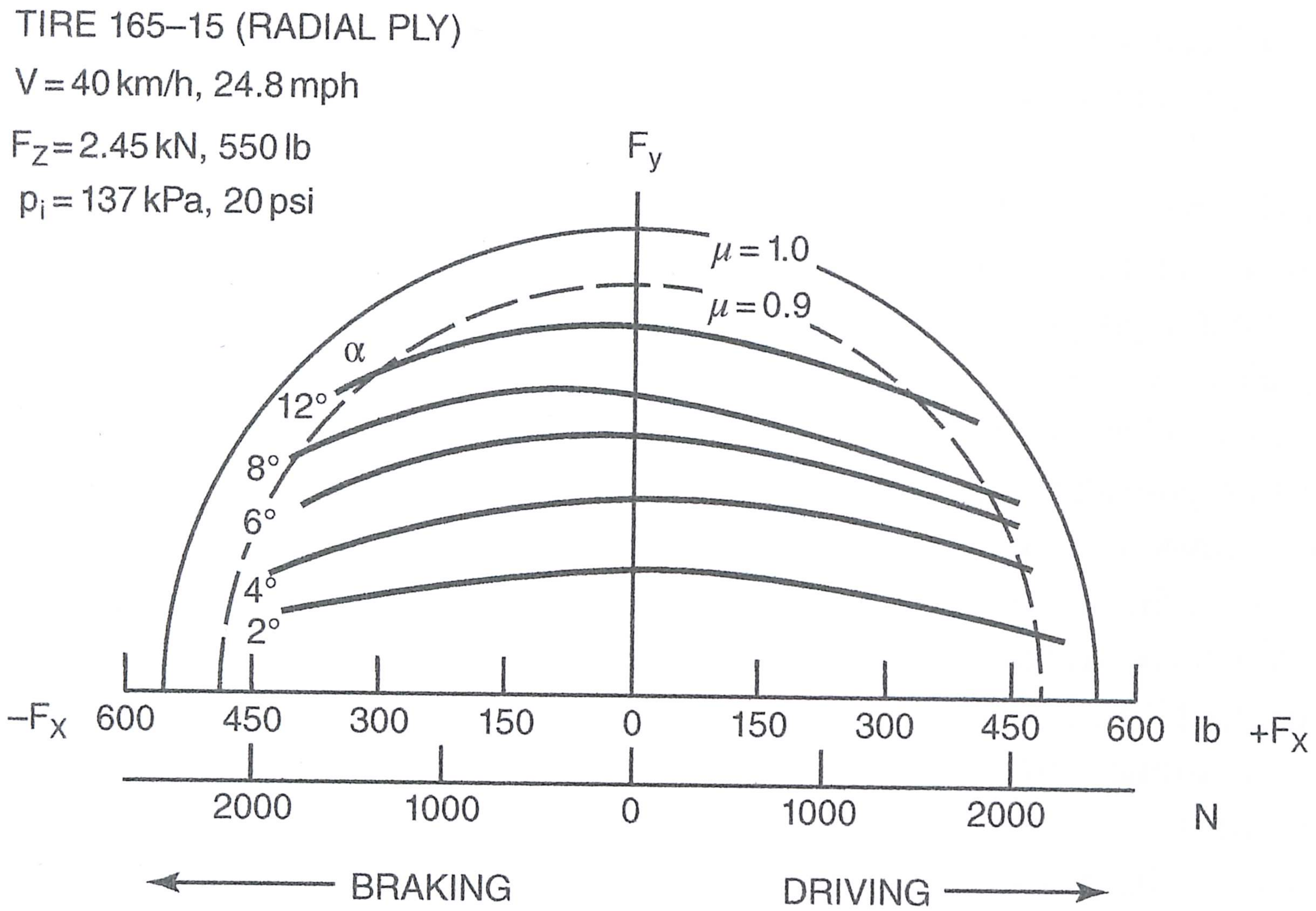
Figur 1.39 visar hur sambandet mellan dessa krafter och avdriftsvinkeln kan se ut.

En enkel modell för sambandet mellan F_x, F_y och α är att anta att kurvorna i figuren är ellipser.

När vi konstruerar ellipserna utgår vi från att följande är känt:

- Sambandet mellan F_y och α i fallet $F_x = 0$.
- F_{xmax} i fallet $F_y = 0$.

Figur 1.39



Friktionsellipsen: Arbetsgång

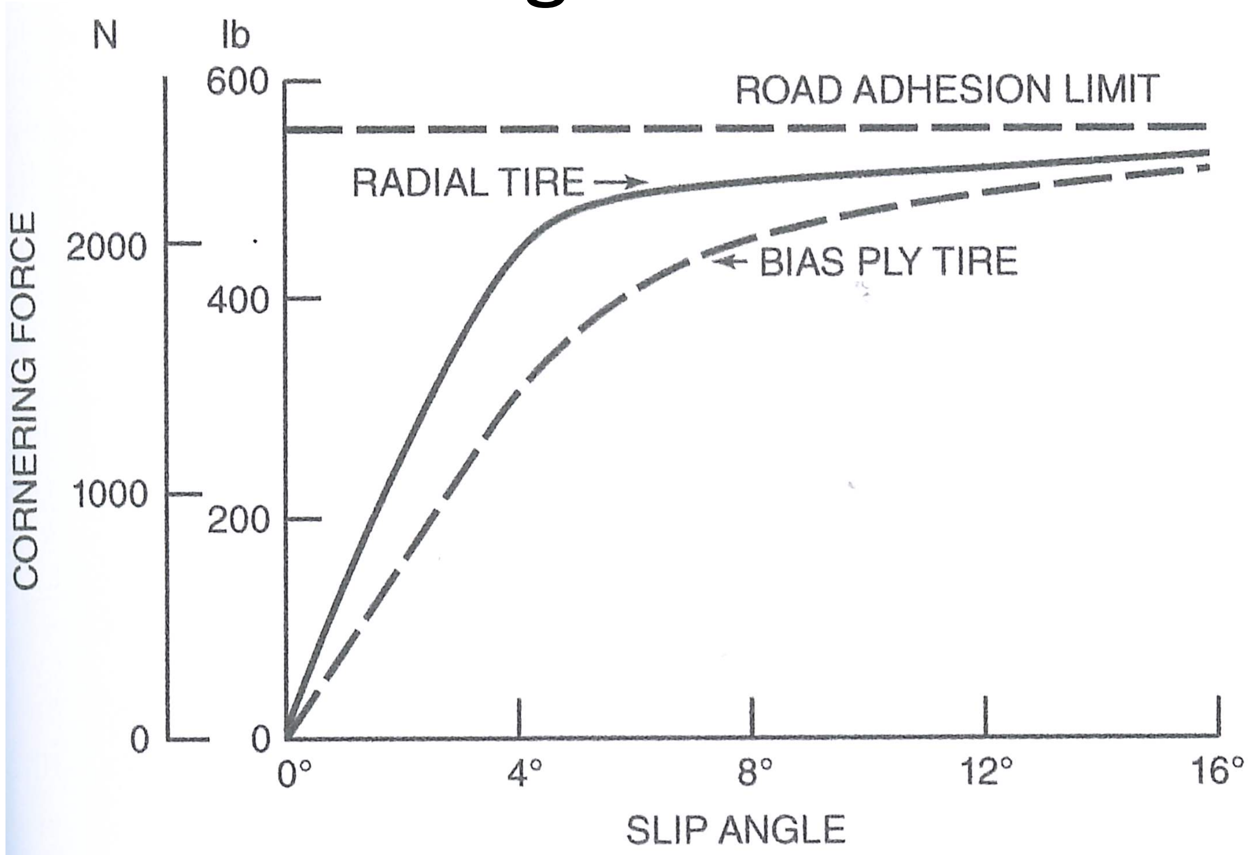
Arbetsgång:

1. Givet en avdriftsvinkel α beräknas $F_{y\alpha}$ då $F_x = 0$,
   t.ex. genom att läsa av figur 1.23 eller motsvarande.
2. Maximala longitudinella kraften F_{xmax} i fallet $F_y = 0$ är känd.
3. F_{xmax} och $F_{y\alpha}$ är halvaxlarna i ellipsen

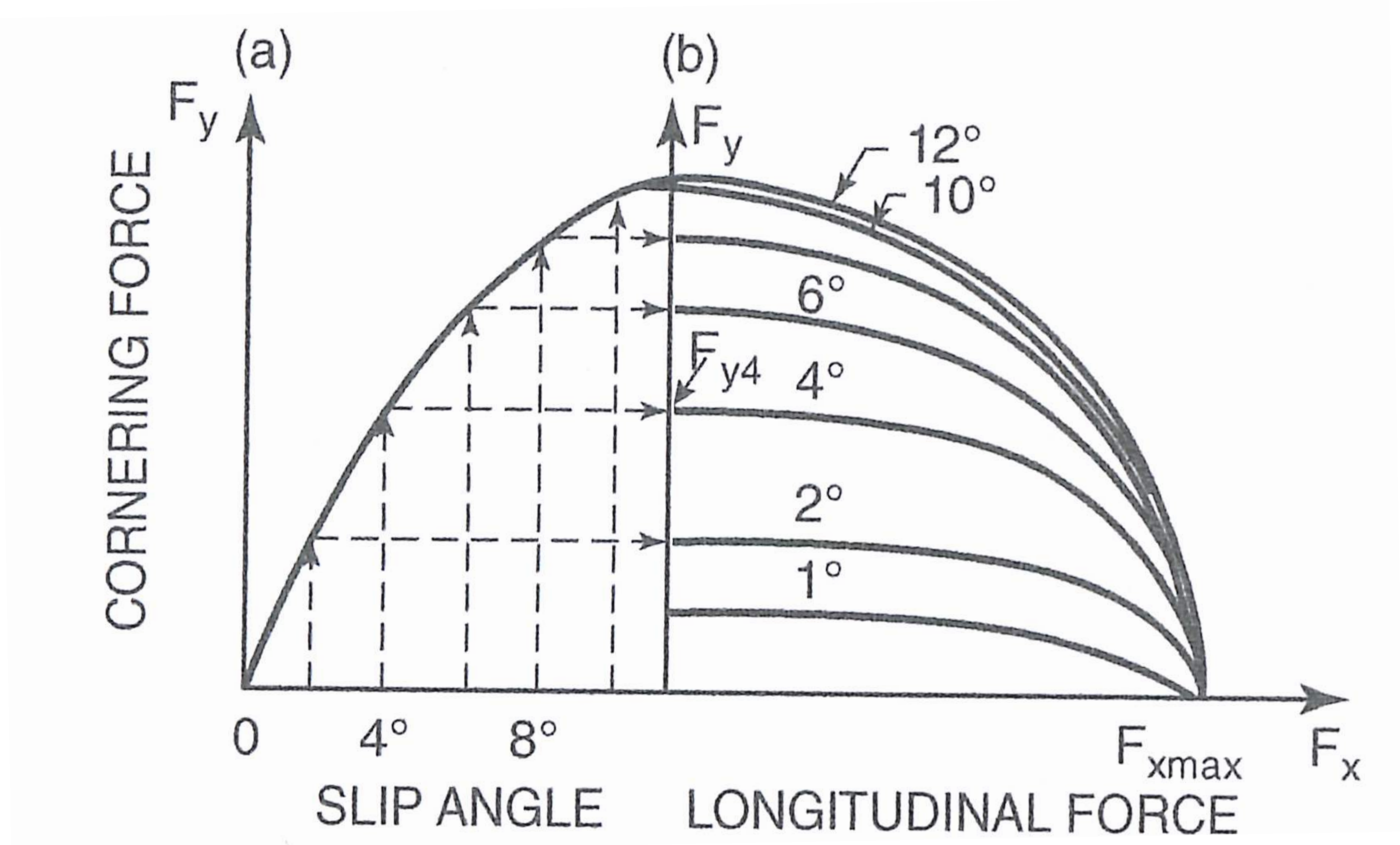
$$\left(\frac{F_y}{F_{y\alpha}}\right)^2 + \left(\frac{F_x}{F_{xmax}}\right)^2 = 1$$

Figur 1.42 illustrerar hur ellipserna ges av F_{xmax} och kurvan $F_y(\alpha)$.

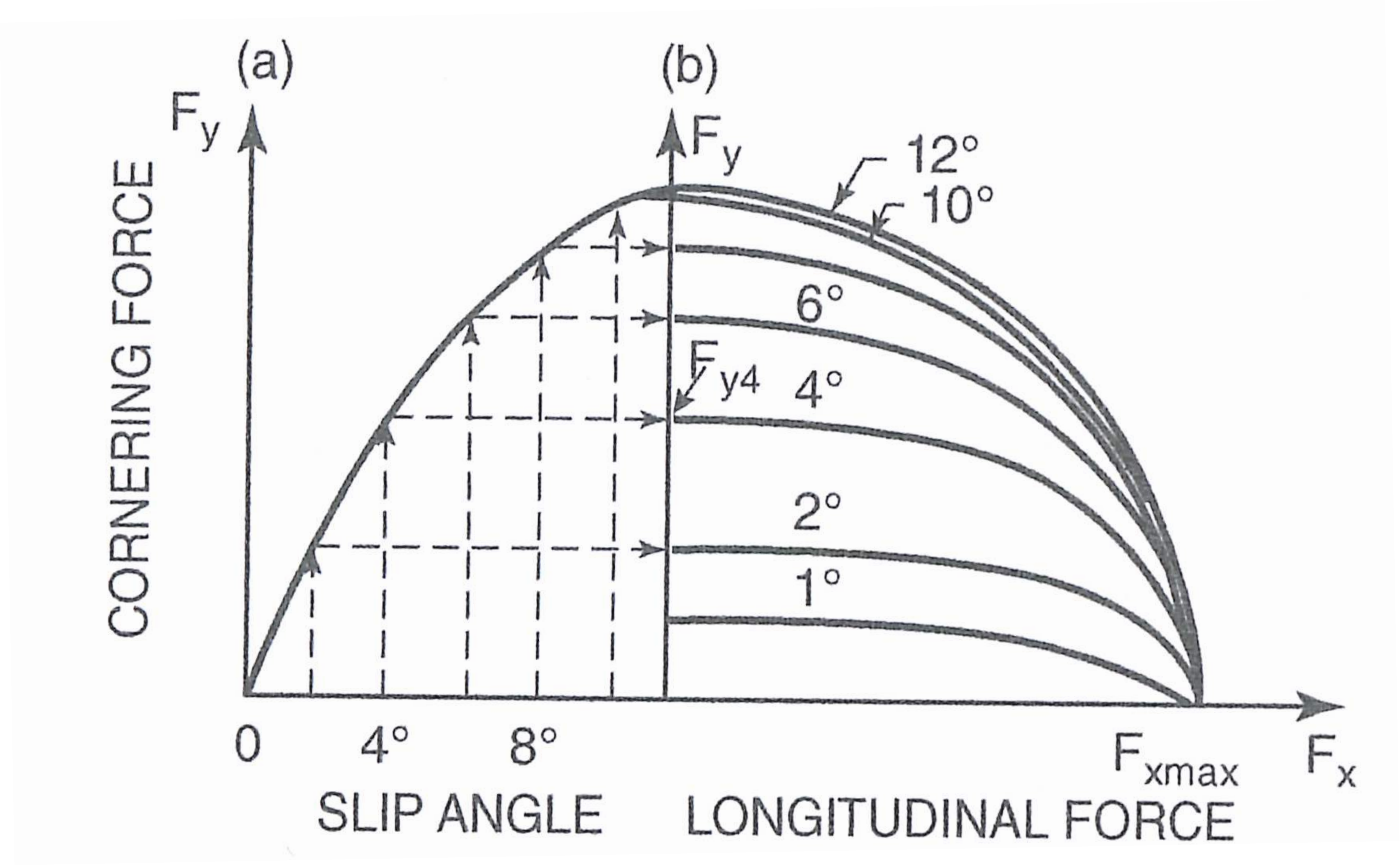
Figur 1.23



Figur 1.42

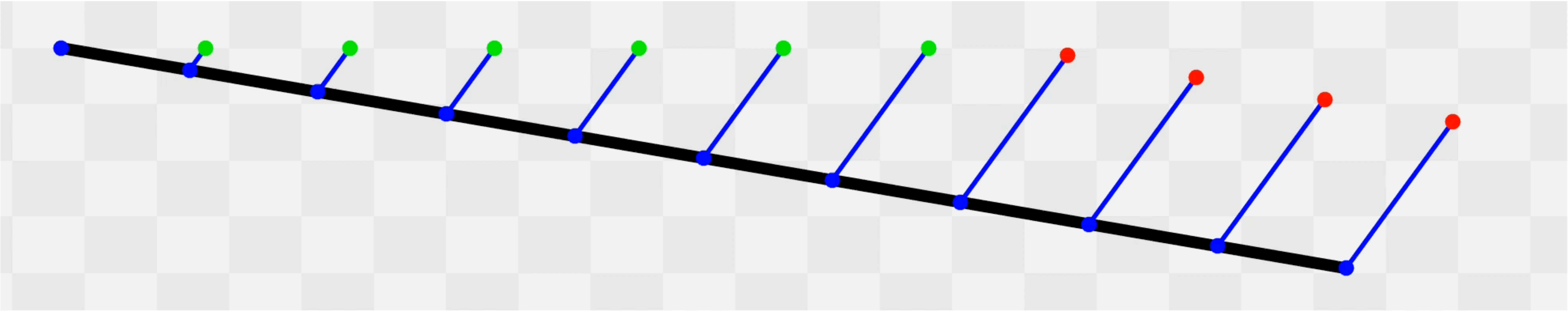


Figur 1.42


(a)
(b)
F_y
CORNERING FORCE
F_y
12°
10°
6°
F_{y4}
4°
2°
1°
0
4°
8°
SLIP ANGLE
F_{xmax}
F_x
LONGITUDINAL FORCE

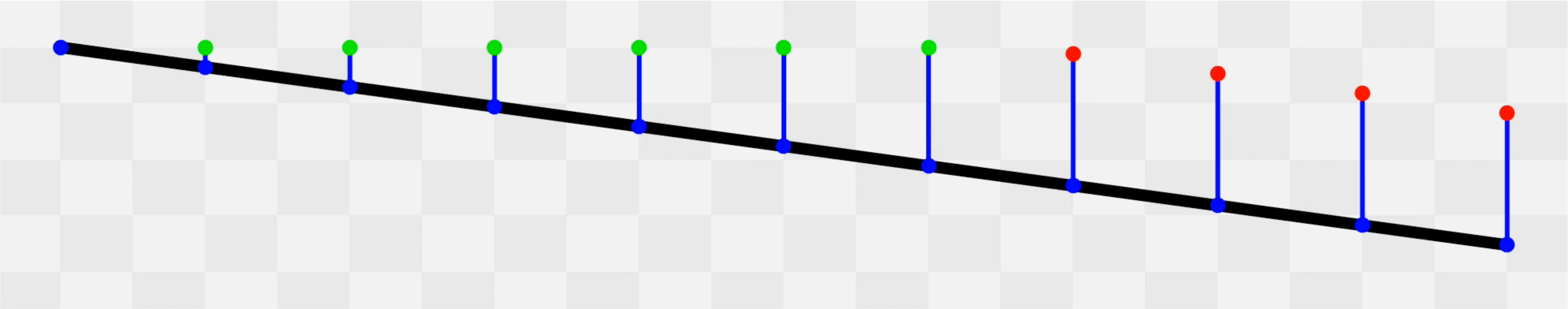
Borstmodellen

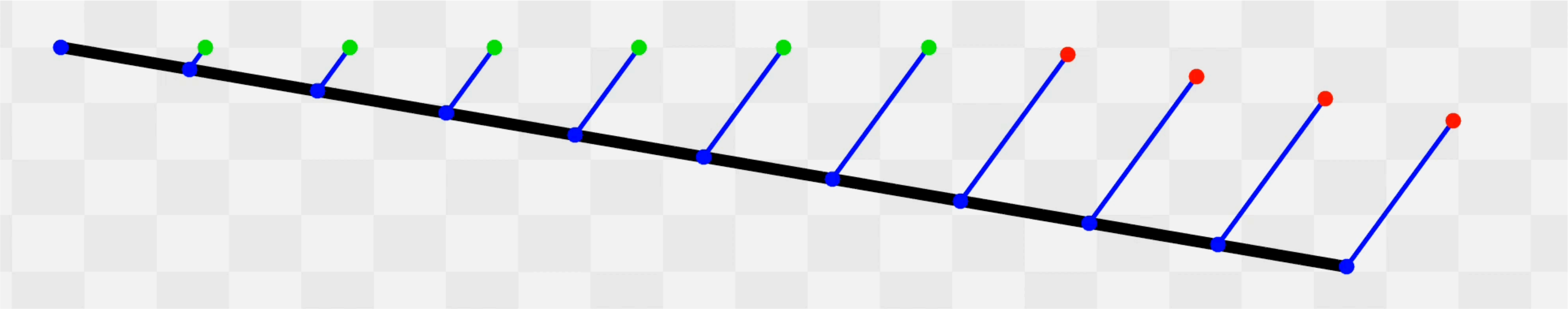
Tidigare har vi använt borstmodellen för lateral och longitudinella krafter separat. Modellen går lätt att utvidga till det allmänna fallet. Borstmodellen sedd uppifrån:



De gröna cirklarna representerar punkter där den nedre delen av strået har kontakt med underlaget utan att glida och de röda där strået har börjat glida.

Borstmodellen: Jämförelse





Borstmodellen: Grundläggande samband

Longitudinell förskjutning

$$e(x) = \frac{i_s}{1 - i_s}x$$

Lateral förskjutning

$$y'(x) = (x + e(x))\tan\alpha \approx \frac{\alpha}{1 - i_s}x$$

Longitudinell kraft med linjär modell:

$$\frac{dF_x}{dx} = \frac{k_t i_s}{1 - i_s}x$$

Lateral kraft med linjär modell

$$\frac{dF_y}{dx} = \frac{k_y' \alpha}{1 - i_s}x$$

Borstmodellen: Grundläggande samband

Modell för normaltrycket (konstant)

$$\frac{dF_z}{dx} = \frac{W}{l_t}$$

Friktionsmodell:

$$\sqrt{\left(\frac{dF_x}{dx}\right)^2 + \left(\frac{dF_y}{dx}\right)^2} \leq \mu \frac{dF_z}{dx}$$

I vilozonen gäller att:

$$\sqrt{\left(\frac{k_t i_s}{1 - i_s}\right)^2 + \left(\frac{k_y' \alpha}{1 - i_s}\right)^2} \cdot x \leq \frac{\mu W}{l_t}$$

Borstmodellen

Längden på vilozonen ges av

$$\frac{l_c}{l_t} = \frac{\mu W(1 - i_s)}{2\sqrt{(C_s i_s)^2 + (C_\alpha \alpha)^2}}$$

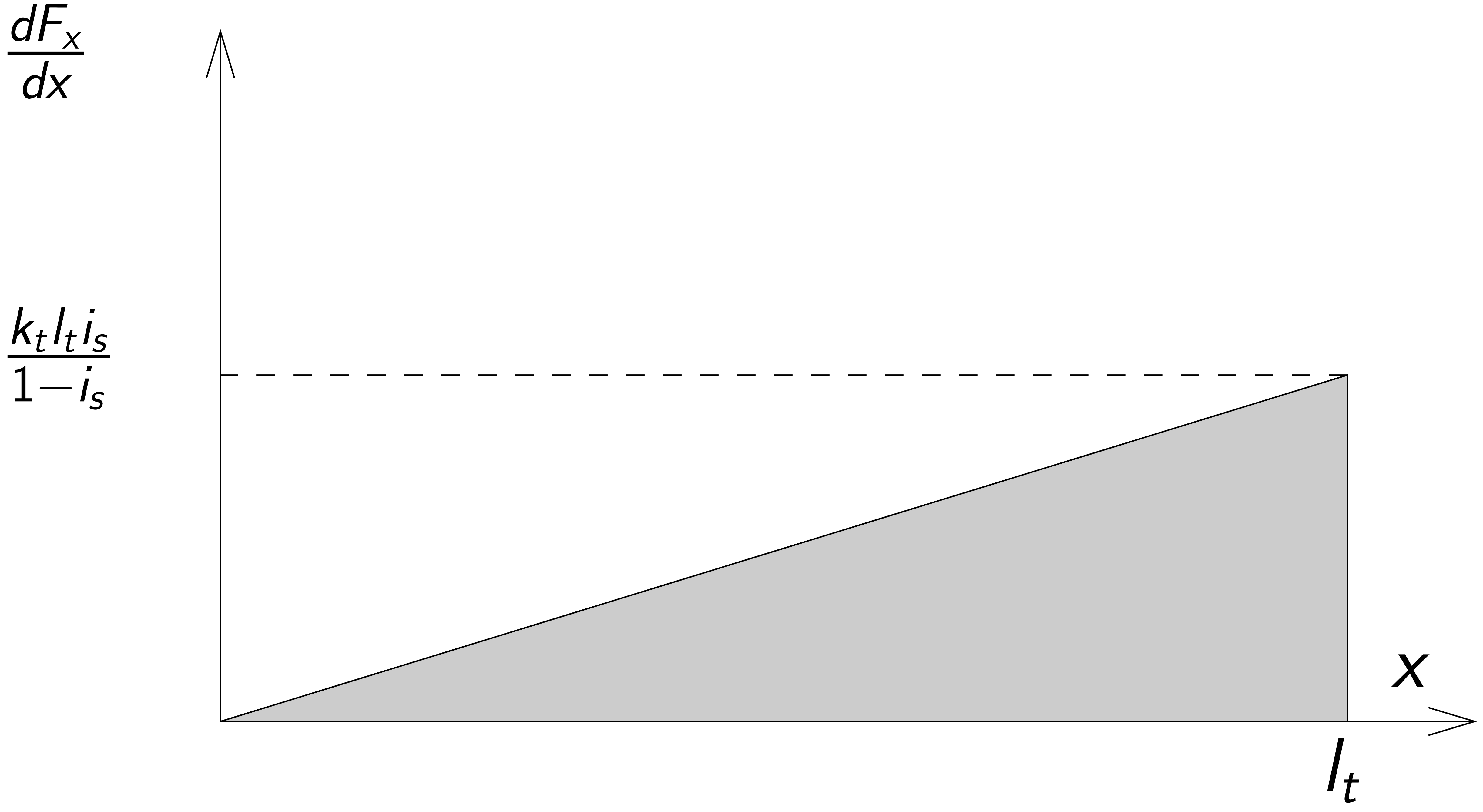
där

$$C_s = \frac{k_t l_t^2}{2}$$

$$C_\alpha = \frac{k_y' l_t^2}{2}$$

Om $l_c/l_t \geq 1$ så finns ingen glidzon

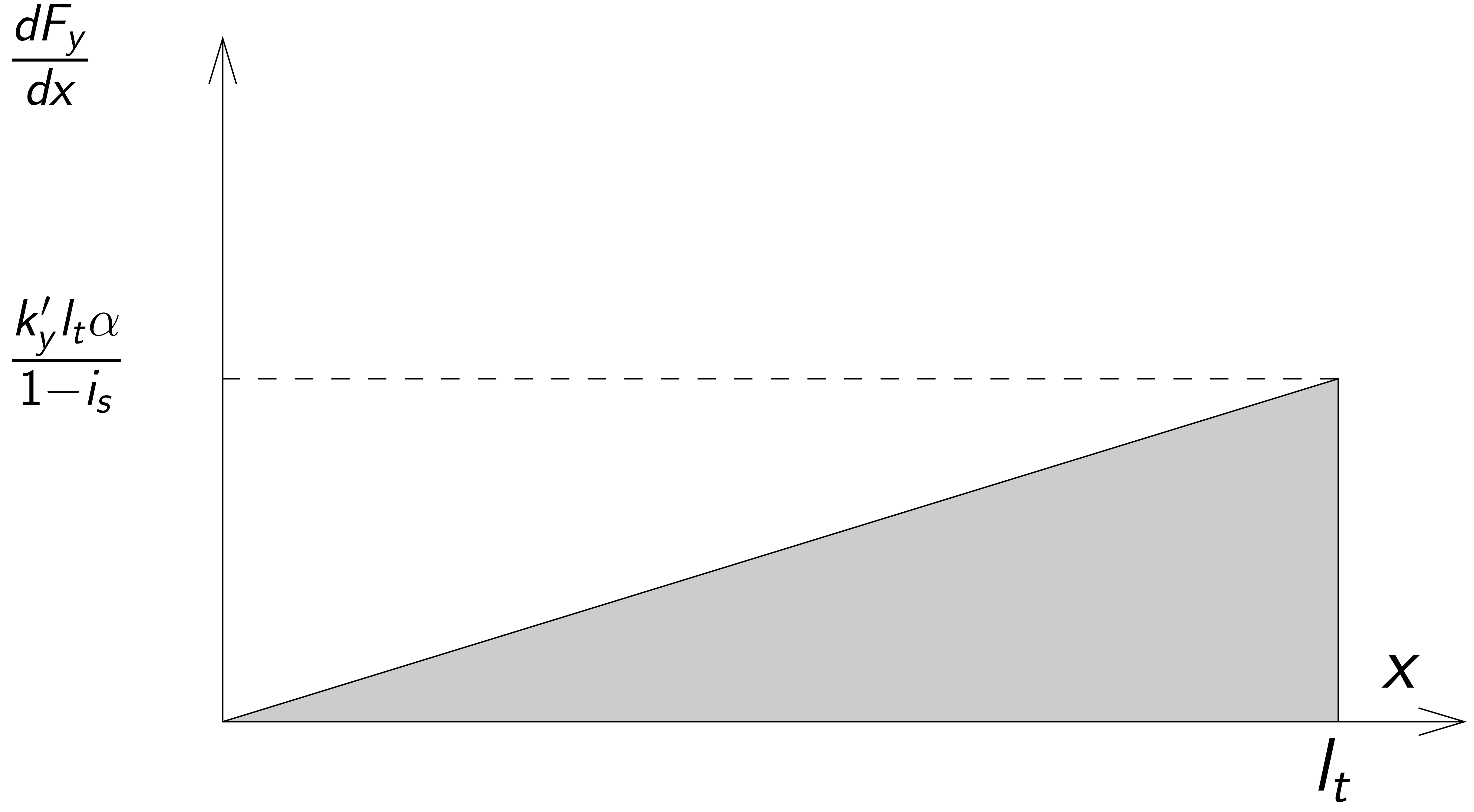
Borstmodellen: Utan glidzon



Den skuggade arean under kurvan ger longitudinella kraften:

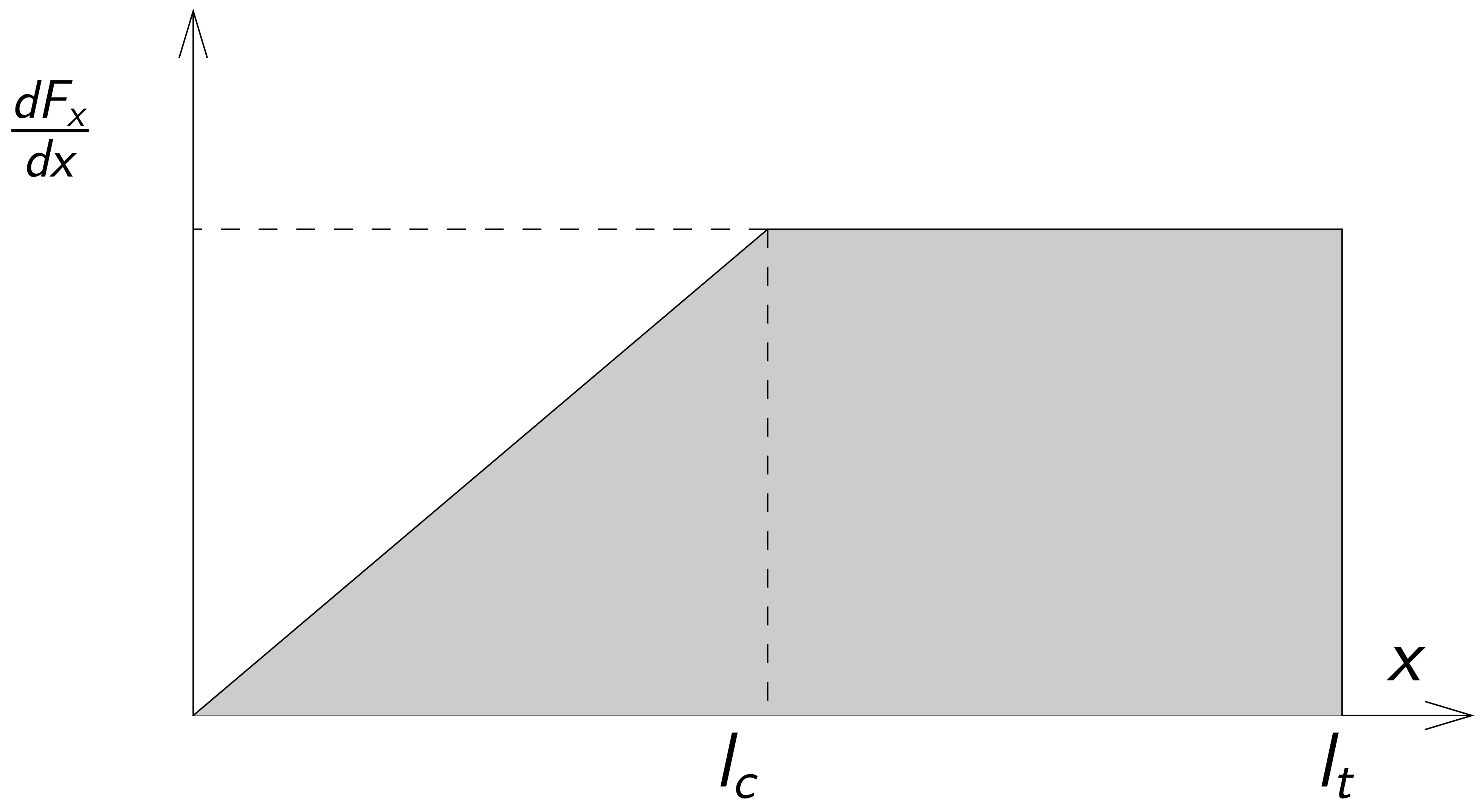
$$F_x = \frac{1}{2}\frac{k_t i_s l_t}{1 - i_s} l_t = C_s \frac{i_s}{1 - i_s}$$

Borstmodellen: Utan glidzon



Arean under kurvan: $F_y = \frac{1}{2}\frac{k_y'\alpha l_t}{1-i_s}l_t = C_\alpha \frac{\alpha}{1-i_s}$

Borstmodellen: Med glidzon

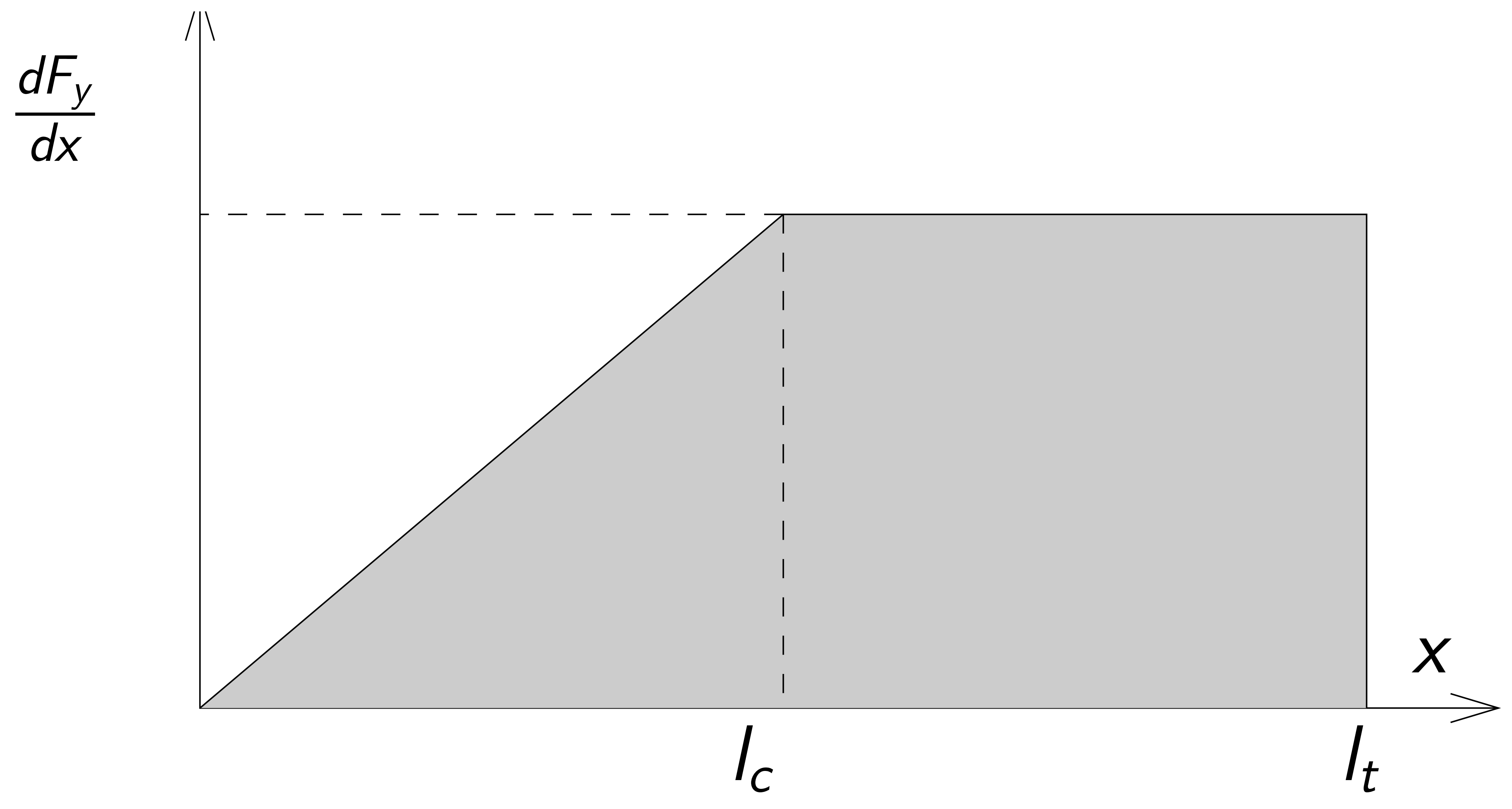


I glidzonen gäller att

$$\frac{dF_x}{dx} = \frac{\mu W}{l_t} \frac{C_s i_s}{\sqrt{(C_s i_s)^2 + (C_\alpha \alpha)^2}}$$

Kraften F_x ges av den skuggade arean under kurvan

Borstmodellen: Med glidzon



I glidzonen gäller att

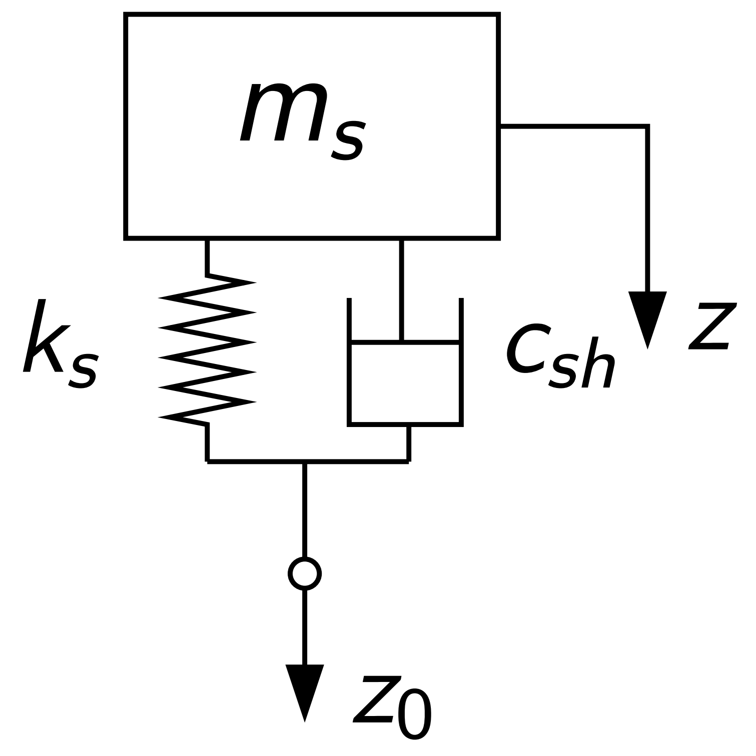
$$\frac{dF_y}{dx} = \frac{\mu W}{l_t} \frac{C_\alpha \alpha}{\sqrt{(C_s i_s)^2 + (C_\alpha \alpha)^2}}$$

Kraften F_y ges av den skuggade arean under kurvan

Vertical Dynamics: The Quarter Car Model

Quarter-car model: Example

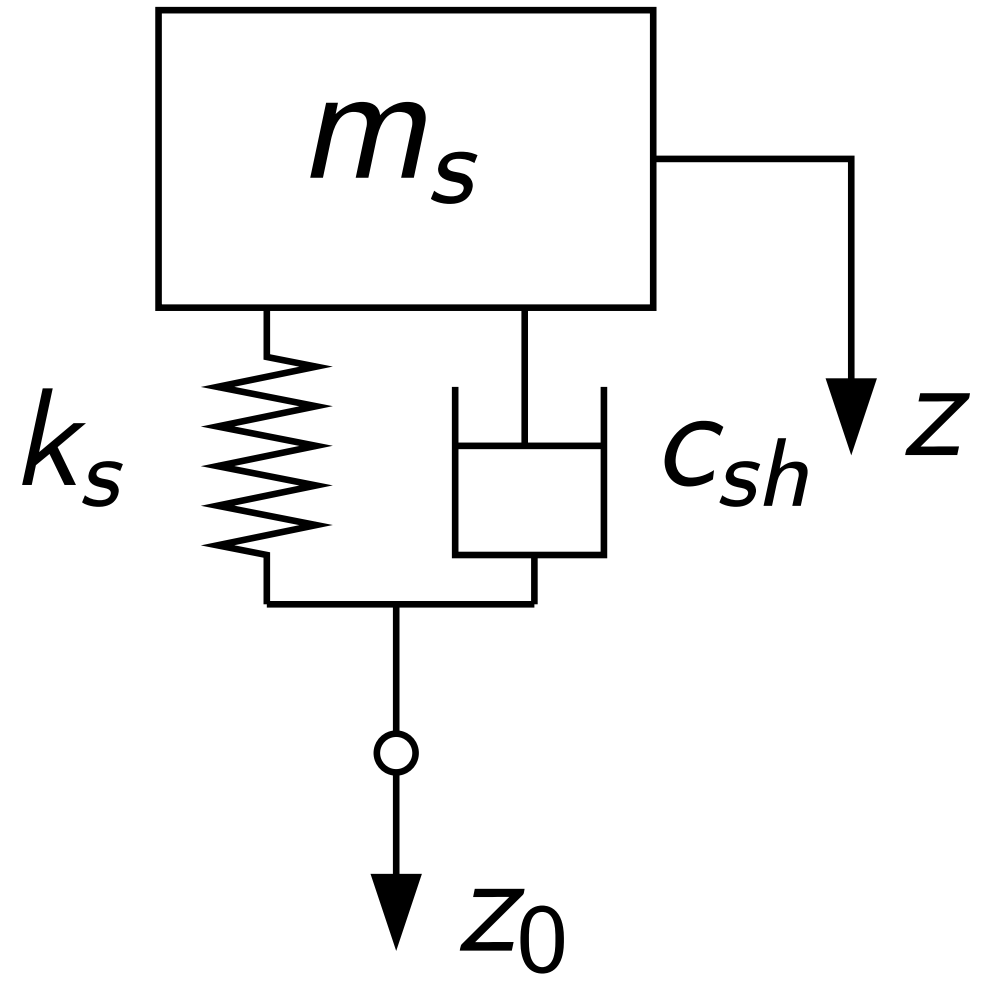
The following quarter-model of a car includes a sprung mass $m_s = 450\ kg$, representing a quarter of the vehicle body, a suspension spring with stiffness $k = 25\ kN/m$, and a shock absorber with damping coefficient $c_{sh} = 2\ kNs/m$:



The coordinate z is the deviation from the static equilibrium position in the vertical direction, and z_0 is the elevation of the surface profile. Assume that the car is traveling at a constant speed $v = 60\ km/h$ on a sinusoidal road with wavelength $\lambda = 20\ m$ and amplitude $\hat{Z} = 5\ cm$. Determine the maximal and minimal force between the tire and the road.

Quarter-car model: Example

- Frequency of the excitation from the road is $\omega = \frac{2\pi v}{\lambda} = 5.34$ rad/s
- Equation of motion: $m_s\ddot{z}(t) = c_{sh}(\dot{z}_0(t) - \dot{z}(t)) + k_s(z_0(t) - z(t))$
- Laplace transform: $s^2 m_s Z(s) = c_{sh}(sZ_0(s) - sZ(s)) + k_s(Z_0(s) - z(S))$
- Transfer function from z_0 to z: $Z(s) = \frac{c_{sh}s + k}{m_s^2 + c_{sh}s + k} Z_0(s)$
- Transfer function from z_0 to $F = m\ddot{z}$: $F(s) = m_s s^2 \frac{c_{sh}s + k}{m_s s^2 + c_{sh}s + k} Z_0(s)$



Quarter-car model: Example

The transfer function from z_0 to the force F is

$$F(s) = \frac{m_s s^2(c_{sh}s + k)}{m_s s^2 + c_{sh}s + k} Z_0(s) = G(s)Z_0(s)$$

and the amplitude of the force is

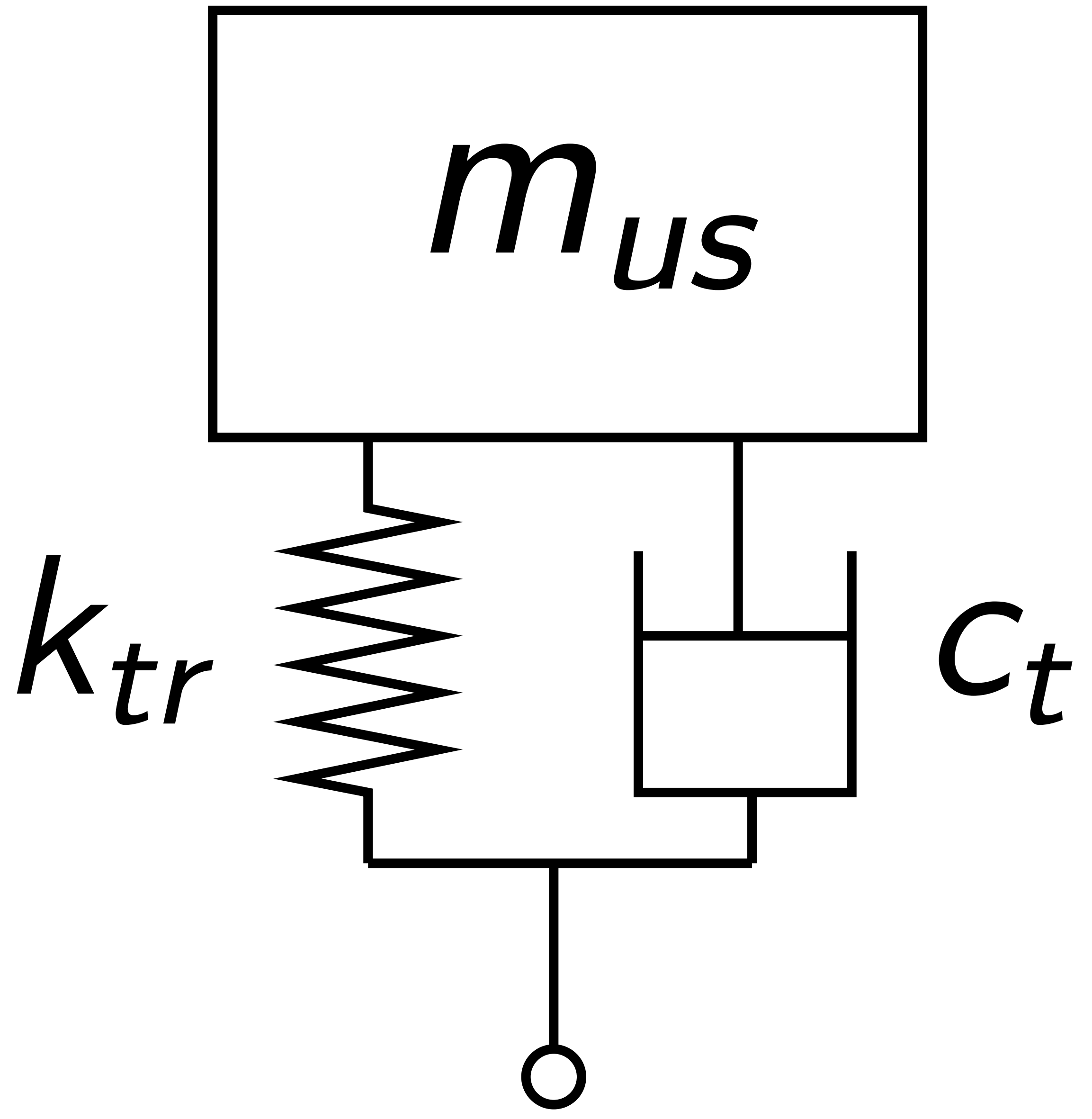
$$|G(i\omega)|\hat{Z} = 1.0 \ kN$$

Maximal/minimal force between the tire and ground

$$mg \pm |G(i\omega)|Z = 4.4 \ kN \pm 1.0 \ kN$$

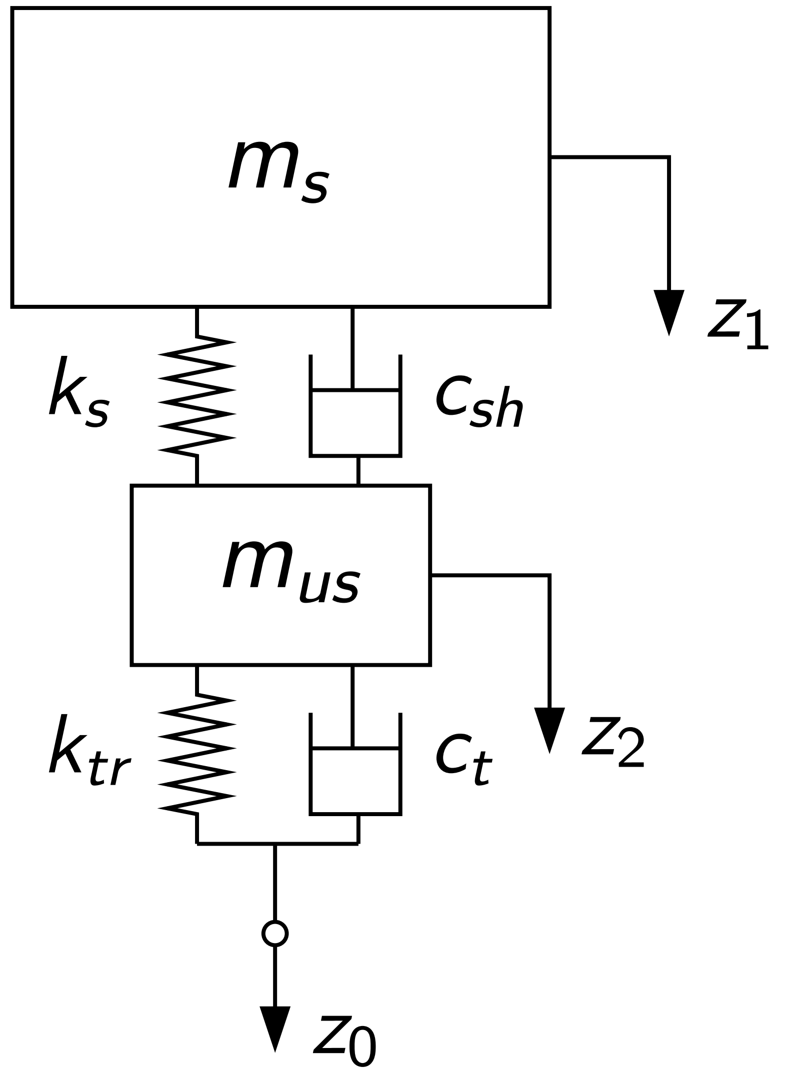
Tire model for vertical motion

The previous model will now be refined by adding a tire model with spring, a damper, and an unsprung mass m_{us} representing the wheel and associated components:



Quarter-car model

The result is the following model



with the corresponding equations of motion:

$$m_s\ddot{z}_1 = c_{sh}(\dot{z}_2 - \dot{z}_1) + k_s(z_2 - z_1)$$

$$m_{us}\ddot{z}_2 = c_{sh}(\dot{z}_1 - \dot{z}_2) + k_s(z_1 - z_2) + c_t(\dot{z}_0 - c_t\dot{z}_2) + k_{tr}(z_0 - z_2)$$

Quarter-car model: Undamped system

First, the case and $z_0 = 0$ and without damping is considered.

In this case, the following remains of the model:

$$m_s\ddot{z}_1 + k_s z_1 - k_s z_2 = 0$$
$$m_{us}\ddot{z}_2 - k_s z_1 + (k_s + k_{tr})z_2 = 0$$

or in matrix form

$$M\ddot{\mathbf{z}} + A\mathbf{z} = 0$$

where the matrices

$$M = \begin{bmatrix} m_s & 0 \\ 0 & m_{us} \end{bmatrix}, \quad A = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_{tr} \end{bmatrix}$$

are symmetric and positive definite.

Quarter-car model: Undamped system

We seek solutions in the form

$$\mathbf{z}(t) = \cos(\omega_n t)\begin{pmatrix}\hat{Z}_1 \\ \hat{Z}_2\end{pmatrix} = \cos(\omega_n t)\,\mathbf{Z}$$

where $\hat{Z}_1$ and $\hat{Z}_2$ are the amplitudes of the masses.

Substitute this into the equations

$$M\ddot{\mathbf{z}} + A\mathbf{z} = -\,\omega_n^2 \cos(\omega_n t)\,M\mathbf{Z} + \cos(\omega_n t)\,A\mathbf{Z} = 0$$

$$\cos(\omega_n t)(-\omega_n^2 M + A)\mathbf{Z} = 0$$

A non-trivial solution exists if and only if the natural frequency ω_n is a solution of the characteristic equation

$$\det(-\omega_n^2 M + A) = 0$$

and the corresponding eigenvector $\mathbf{Z}$ is the solution of the homogeneous systems of equations

$$(-\omega_n^2 M + A)\mathbf{Z} = 0$$

Quarter-car model: Undamped system

The solutions of the characteristic equation are

$$\omega_n^2 = \frac{B_1 \pm \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

where

$$A_1 = m_s m_{us}$$
$$B_1 = m_s k_s + m_s k_{tr} + m_{us} k_s$$
$$C_1 = k_s k_{tr}$$

The constants k_s, m_{us} are usually small compared to k_{tr}, m_s.

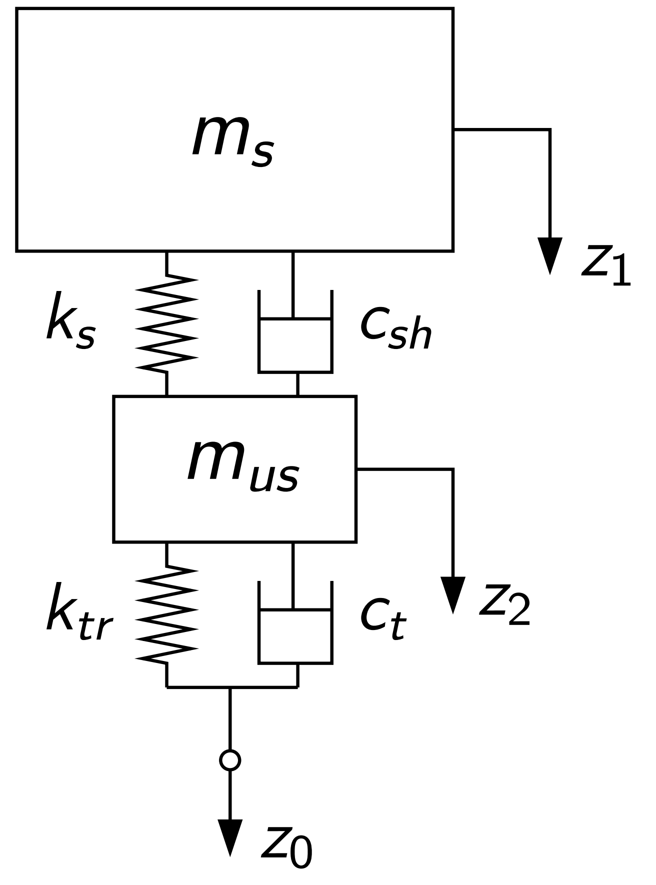
Quarter-car model: Undamped system

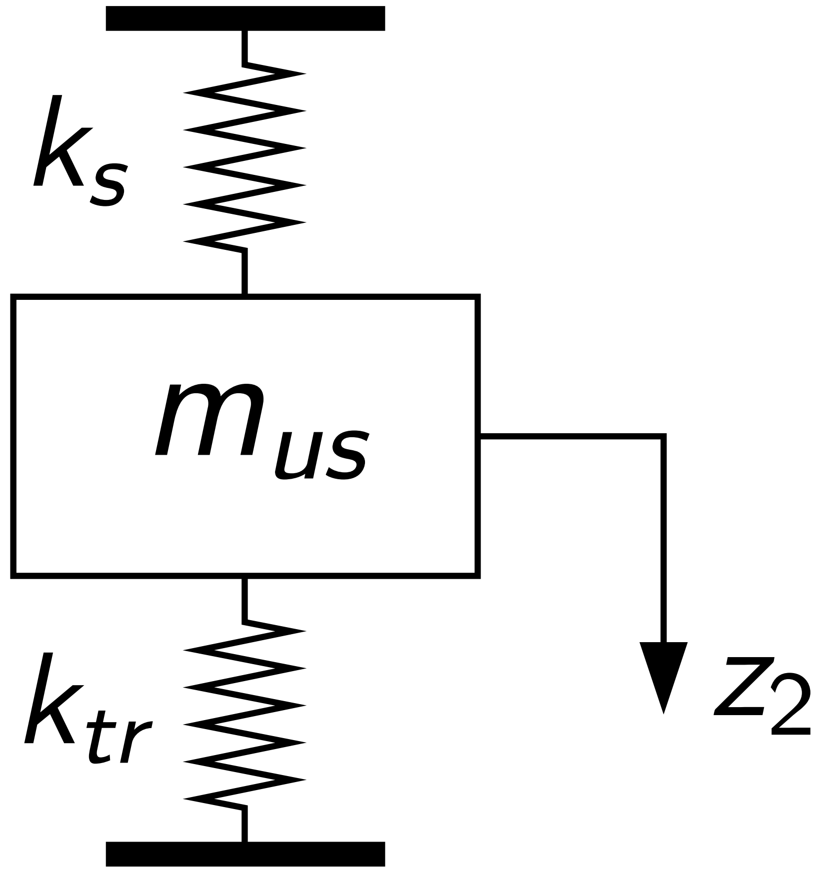
An approximation of the first natural frequency is

$$\omega_{n1}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \approx \frac{B_1}{A_1}$$

$$\approx \frac{m_s k_s + m_s k_{tr}}{m_s m_{us}} = \frac{k_s + k_{tr}}{m_{us}}$$

The approximation is the natural frequency of a system with a mass m_{us} and two springs in parallel.





$A_1 = m_s m_{us}$

$B_1 = m_s k_s + m_s k_{tr} + m_{us} k_s$

$C_1 = k_s k_{tr}$

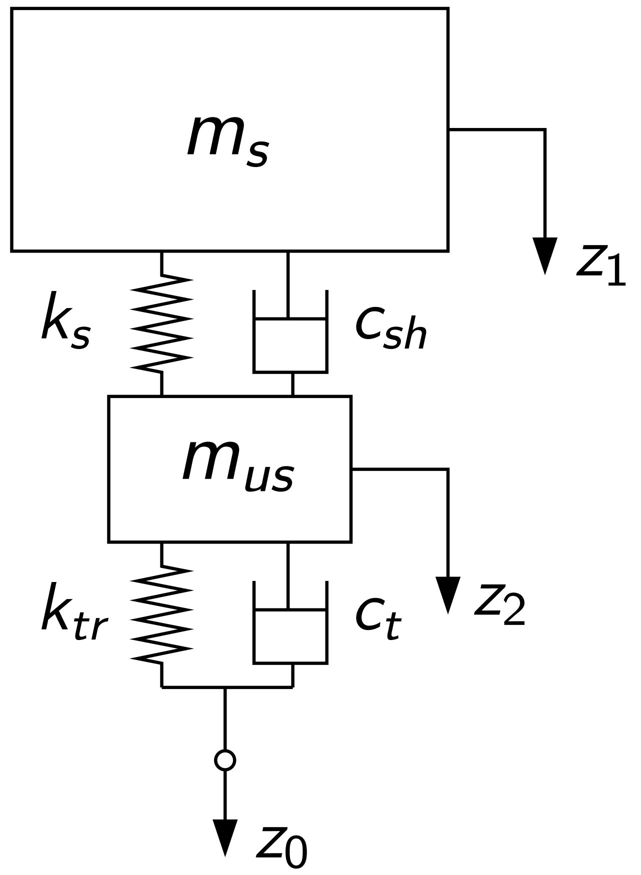
where $k_s \ll k_{tr}, m_{us} \ll m_s$

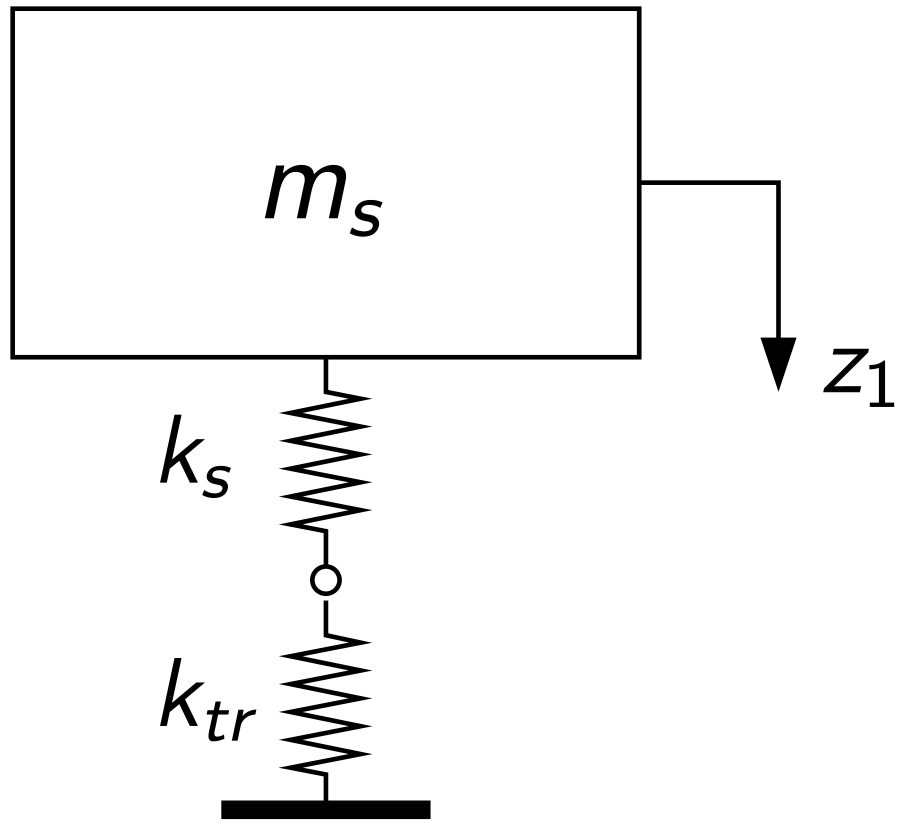
Quarter-car model: Undamped system

An approximation of the second natural frequency is

$$\omega_{n2}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1} = \frac{2C_1}{B_1 + \sqrt{B_1^2 - 4A_1C_1}}$$

$$\approx \frac{C_1}{B_1} \approx \frac{k_s k_{tr}}{m_s k_s + m_s k_{tr}} = \frac{(1/k_s + 1/k_{tr})^{-1}}{m_s}$$





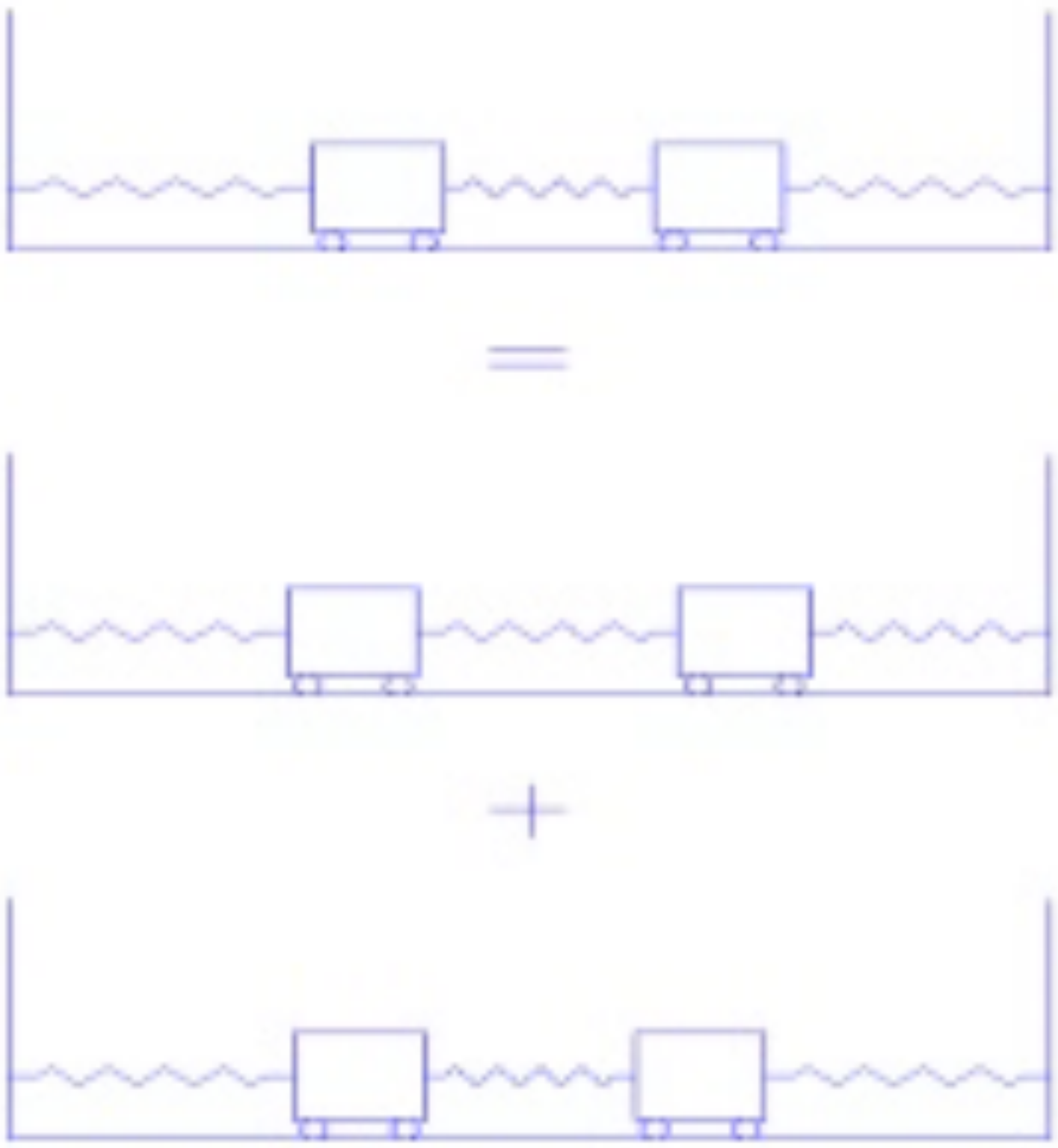
The approximation is the natural frequency of a system with a mass m_s and two springs in series:

$A_1 = m_s m_{us}$

$B_1 = m_s k_s + m_s k_{tr} + m_{us} k_s$

$C_1 = k_s k_{tr}$

where $k_s \ll k_{tr}, m_{us} \ll m_s$



Quarter-car model: Damped system

The model with damping coefficients c_t, c_{sh}, and road profile z_0 included is

$$m_s\ddot{z}_1 = c_{sh}(\dot{z}_2 - \dot{z}_1) + k_s(z_2 - z_1)$$

$$m_{us}\ddot{z}_2 = c_{sh}(\dot{z}_1 - \dot{z}_2) + k_s(z_1 - z_2) + c_t(\dot{z}_0 - c_t\dot{z}_2) + k_{tr}(z_0 - z_2)$$

or

$$M\ddot{\mathbf{z}} + C\dot{\mathbf{z}} + A\mathbf{z} = \mathbf{f}(t)$$

where

$$C = \begin{bmatrix} c_{sh} & -c_{sh} \\ -c_{sh} & c_{sh} + c_t \end{bmatrix}, \quad \mathbf{f}(t) = \begin{pmatrix} 0 \\ c_t\dot{z}_0 + k_{tr}z_0 \end{pmatrix}$$

By taking the Laplace transform of the system we get

$G(s)\mathbf{Z}(s) = \mathbf{F}(s)$ where $G(s) = s^2M + sC + A$

Quarter-car model: Transfer functions

By neglecting the damping in the tire, i.e., $c_t = 0$, and inverting G, with

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

and

$$\mathbf{F} = \begin{pmatrix} 0 \\ k_{tr} Z_0(s) \end{pmatrix}$$

we get the transfer functions $G_{01}(s)$ and $G_{02}(s)$ from Z_0 to Z_1 and Z_2:

$$\begin{aligned} \begin{pmatrix} Z_1(s) \\ Z_2(s) \end{pmatrix} &= \frac{1}{\det G} \begin{bmatrix} g_{22}(s) & -g_{12}(s) \\ -g_{21}(s) & g_{11}(s) \end{bmatrix} \begin{pmatrix} 0 \\ k_{tr} Z_0(s) \end{pmatrix} \\ &= \frac{k_{tr}}{g_{11}(s) g_{22}(s) - g_{12}(s) g_{21}(s)} \begin{pmatrix} -g_{12}(s) Z_0(s) \\ g_{11}(s) Z_0(s) \end{pmatrix} = \begin{pmatrix} G_{01}(s) \\ G_{02}(s) \end{pmatrix} Z_0(s) \end{aligned}$$

Quarter-car model: Amplitude gains

If the excitation $z_0(t)$ from the road is a sinusoidal function with amplitude with angular frequency ω, then the amplitude gains are

$$\frac{\hat{Z}_1}{\hat{Z}_0} = |G_{01}(i\omega)| = \left| \frac{k_{tr} g_{12}(i\omega)}{g_{11}(i\omega) g_{22}(i\omega) - g_{12}(i\omega) g_{21}(i\omega)} \right|$$

$$\frac{\hat{Z}_2}{\hat{Z}_0} = |G_{02}(i\omega)| = \left| \frac{k_{tr} g_{11}(i\omega)}{g_{11}(i\omega) g_{22}(i\omega) - g_{12}(i\omega) g_{21}(i\omega)} \right|$$

where $\hat{Z}_1$ and $\hat{Z}_2$ denote the amplitudes of z_1 and z_2, respectively.

Recall that $G(s) = s^2 M + sC + K$

Quarter-car model: Amplitude gains

It follows from the definition $G(s) = s^2M + sC + K$, after some tedious calculations, that

$$\frac{\hat{Z}_1}{\hat{Z}_0} = \sqrt{\frac{A_2}{B_2 + C_2}}$$

$$\frac{\hat{Z}_2}{\hat{Z}_0} = \sqrt{\frac{A_3}{B_2 + C_2}}$$

where

$$A_2 = (k_s k_{tr})^2 + (c_{sh} k_{tr} \omega)^2$$

$$A_3 = (k_{tr}(k_s - m_s\omega^2))^2 + (c_{sh} k_{tr} \omega)^2$$

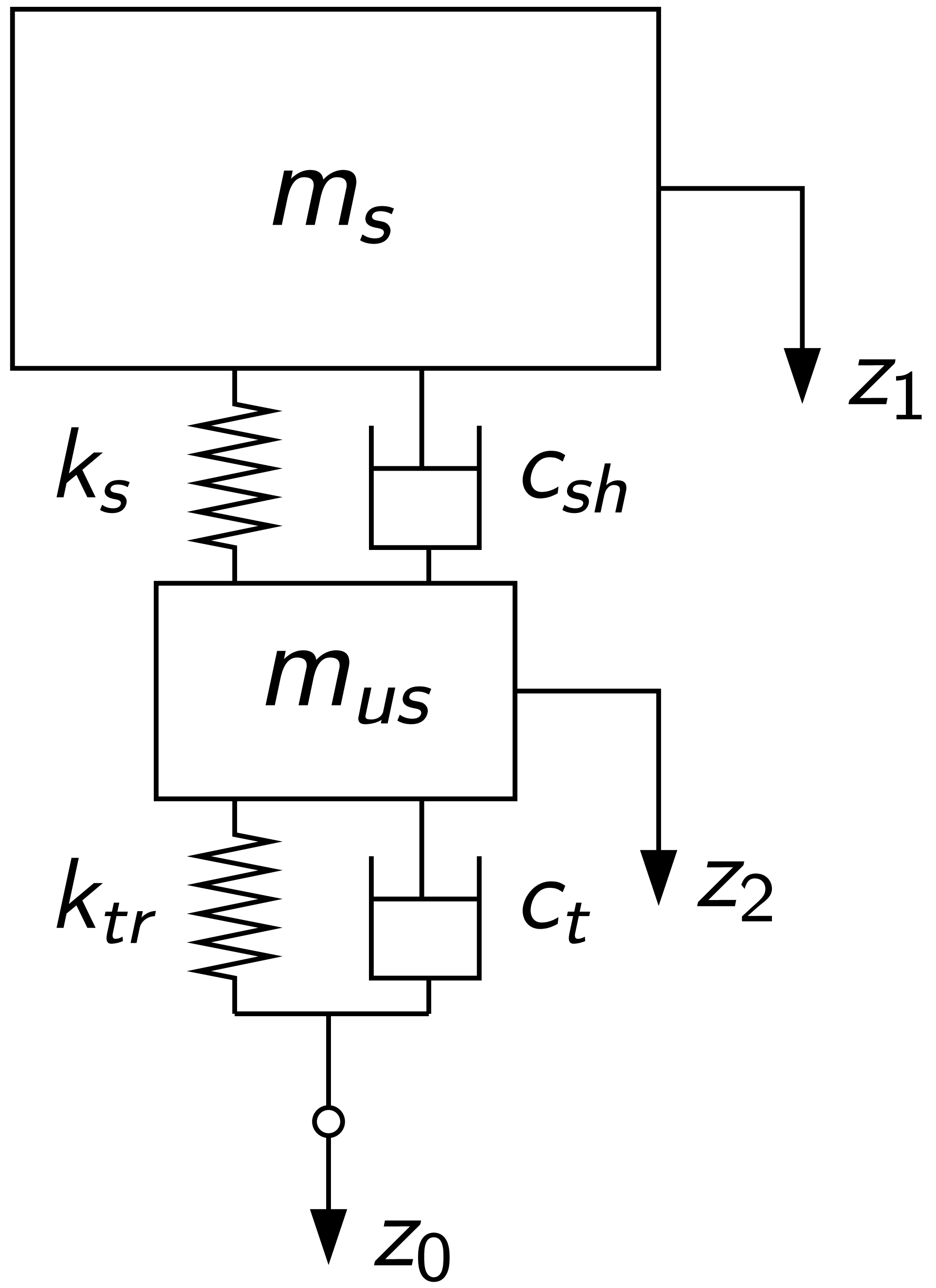
$$B_2 = ((k_s - m_s\omega^2))(k_{tr} - m_{us}\omega^2) - m_s k_s \omega^2)^2$$

$$C_2 = (c_{sh})^2(m_s\omega^2 + m_{us}\omega^2 - k_{tr})^2$$

Quarter-car model: Vibration isolation

Figure 7.9—7.11 in the book show the bode plot for the transfer function from z_0 to z_1, i.e., G_{01}, for different values of the

- Unsprung mass m_{us}
- Stiffness coefficient for the suspension spring k_s
- Damping coefficient γ



Quarter-car model: Vibration isolation

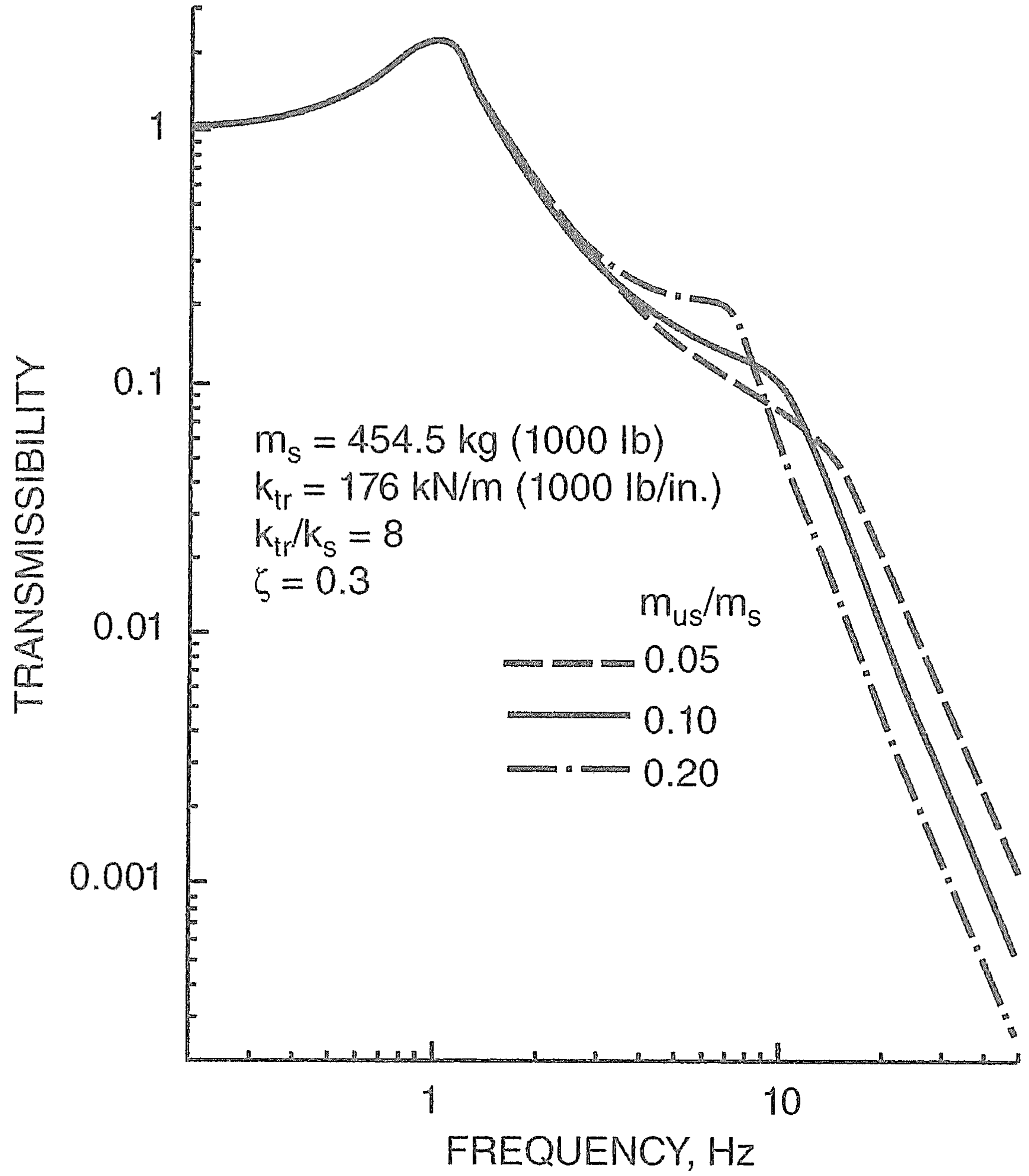


Figure 7.9

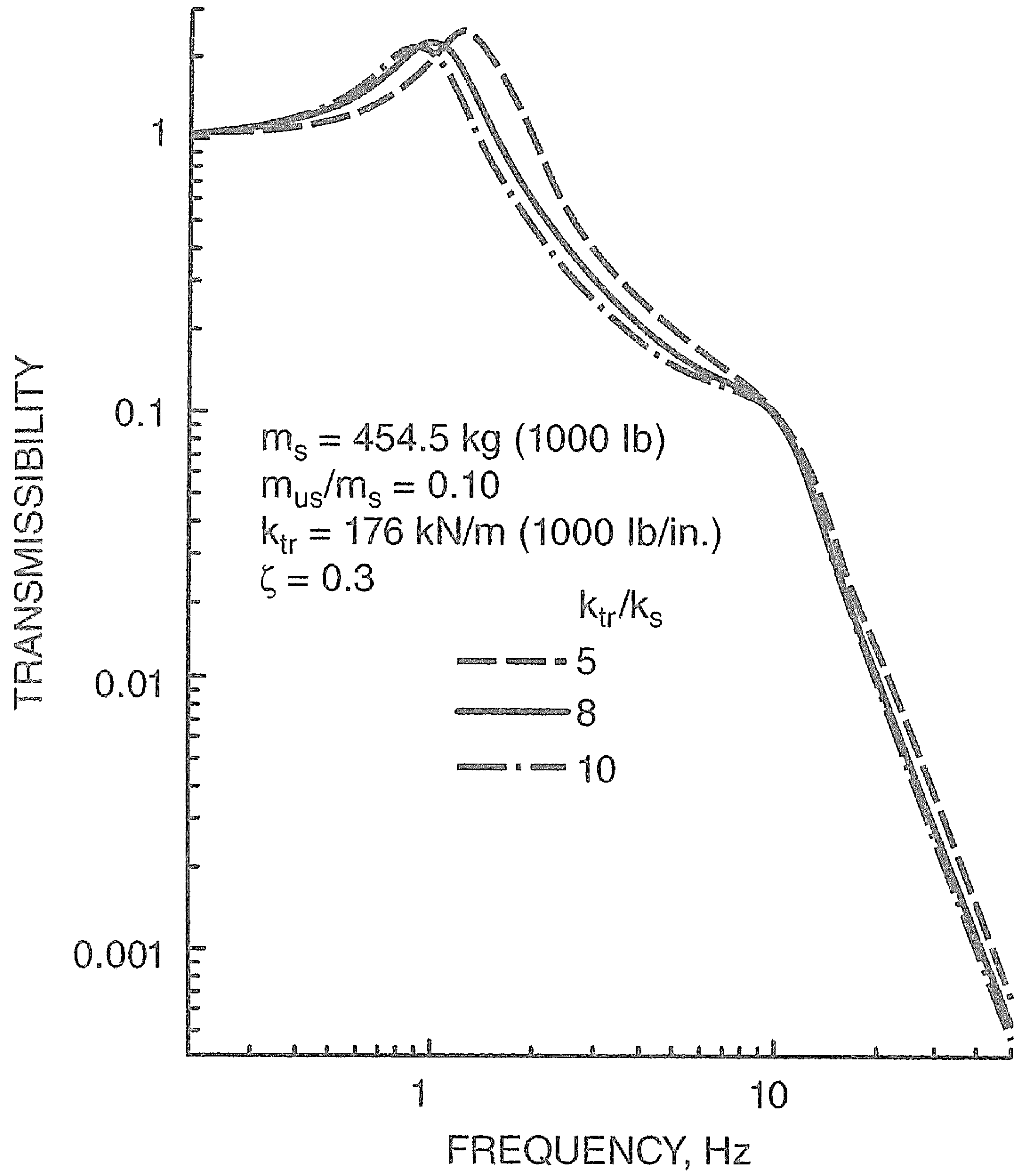


Figure 7.10

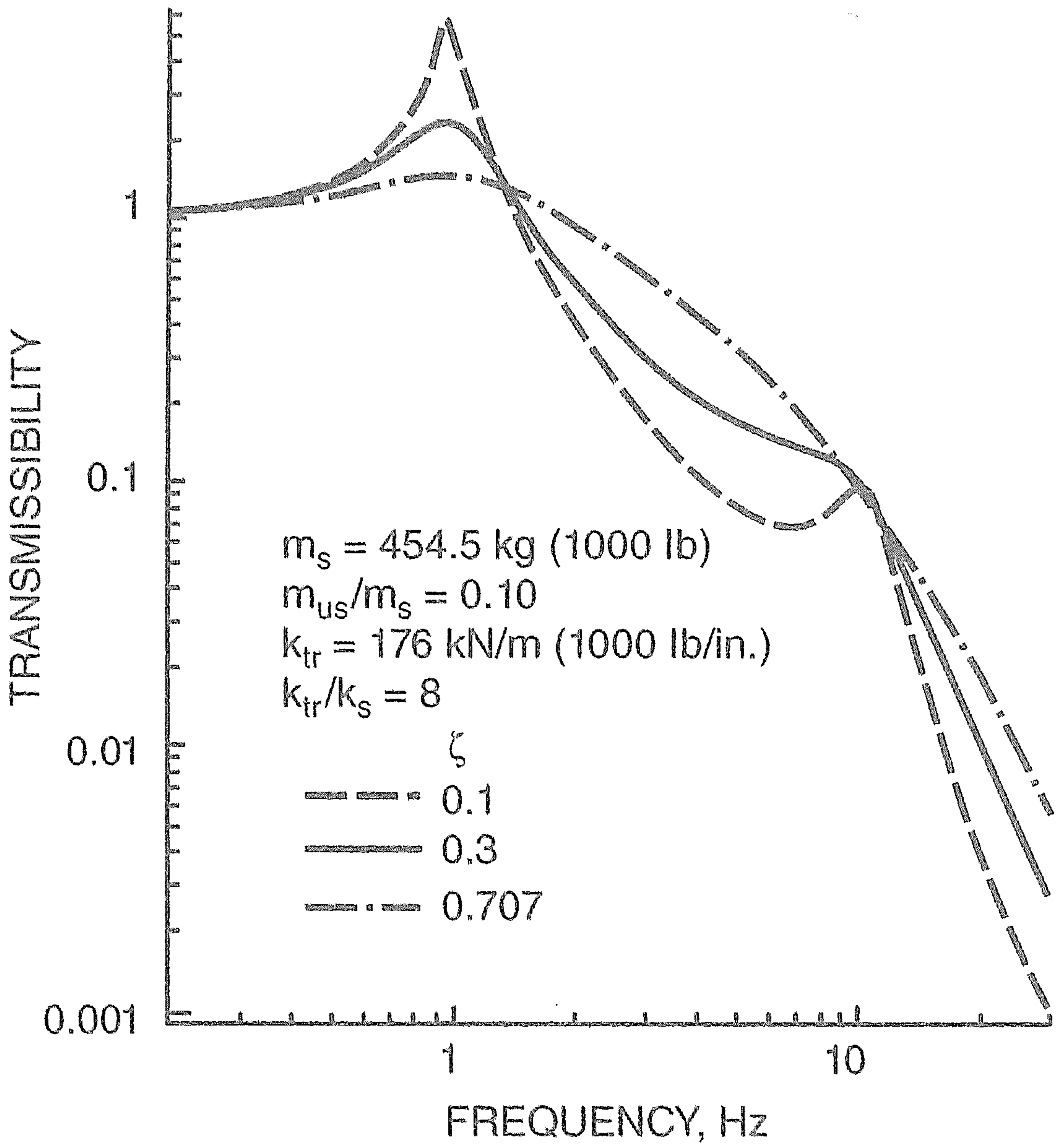


Figure 7.11

Quarter-car model: Suspension travel

The difference $z_2 - z_1$ is the relative displacement between the sprung and unsprung mass, and the amplitude gain

$$\frac{\max(z_2 - z_1)}{\hat{Z}_0} = |G_{02}(i\omega) - G_{01}(i\omega)|$$

influences the required space for the deflection of the suspension.

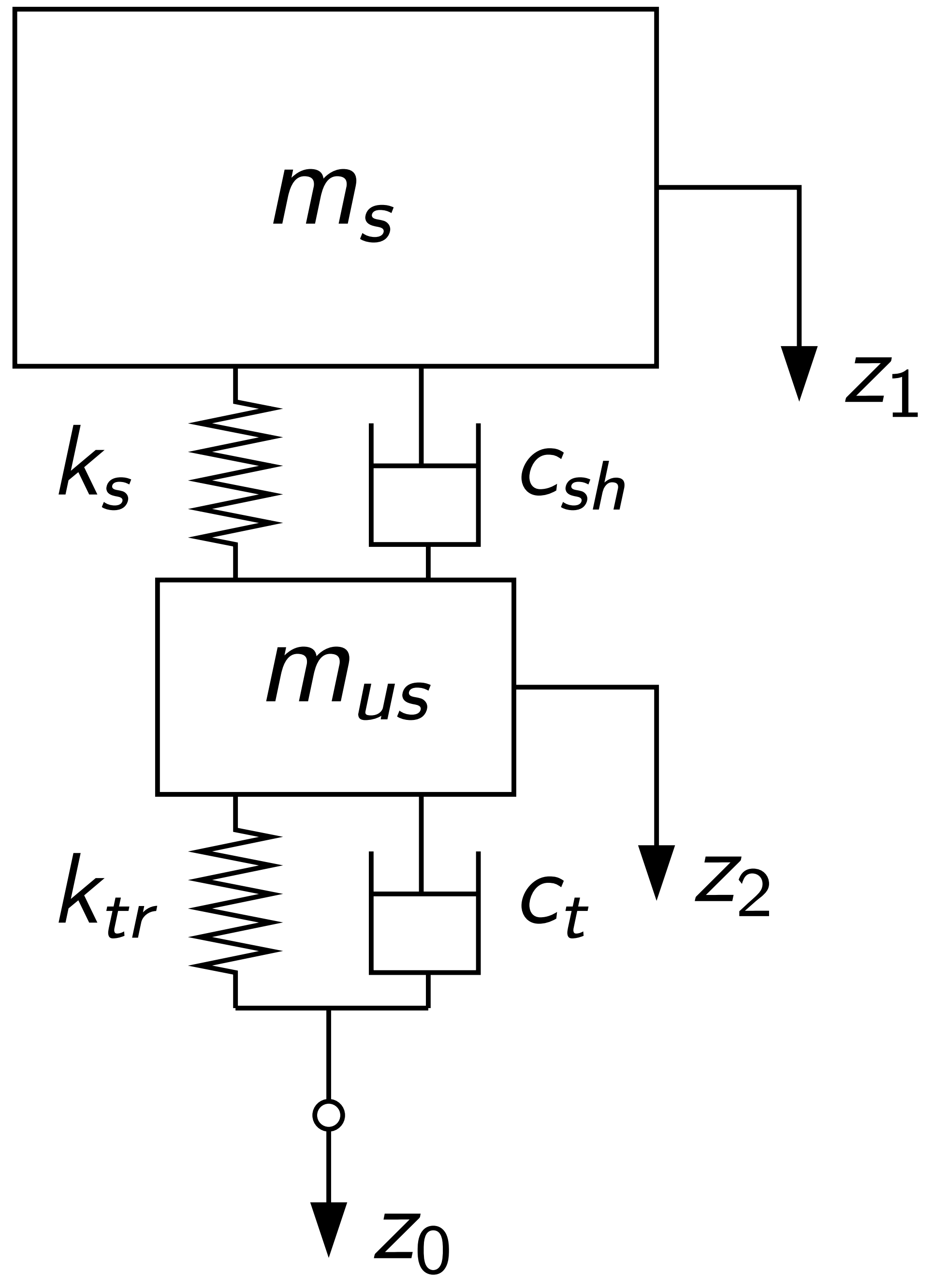


Figure 7.12–7.14 in the book show the bode plot for the transfer function from z_0 to $z_2 - z_1$ for different values of the

- Unsprung mass m_{us}
- Stiffness coefficient of the suspension spring k_s
- Damping coefficient γ

Quarter-car model: Suspension travel

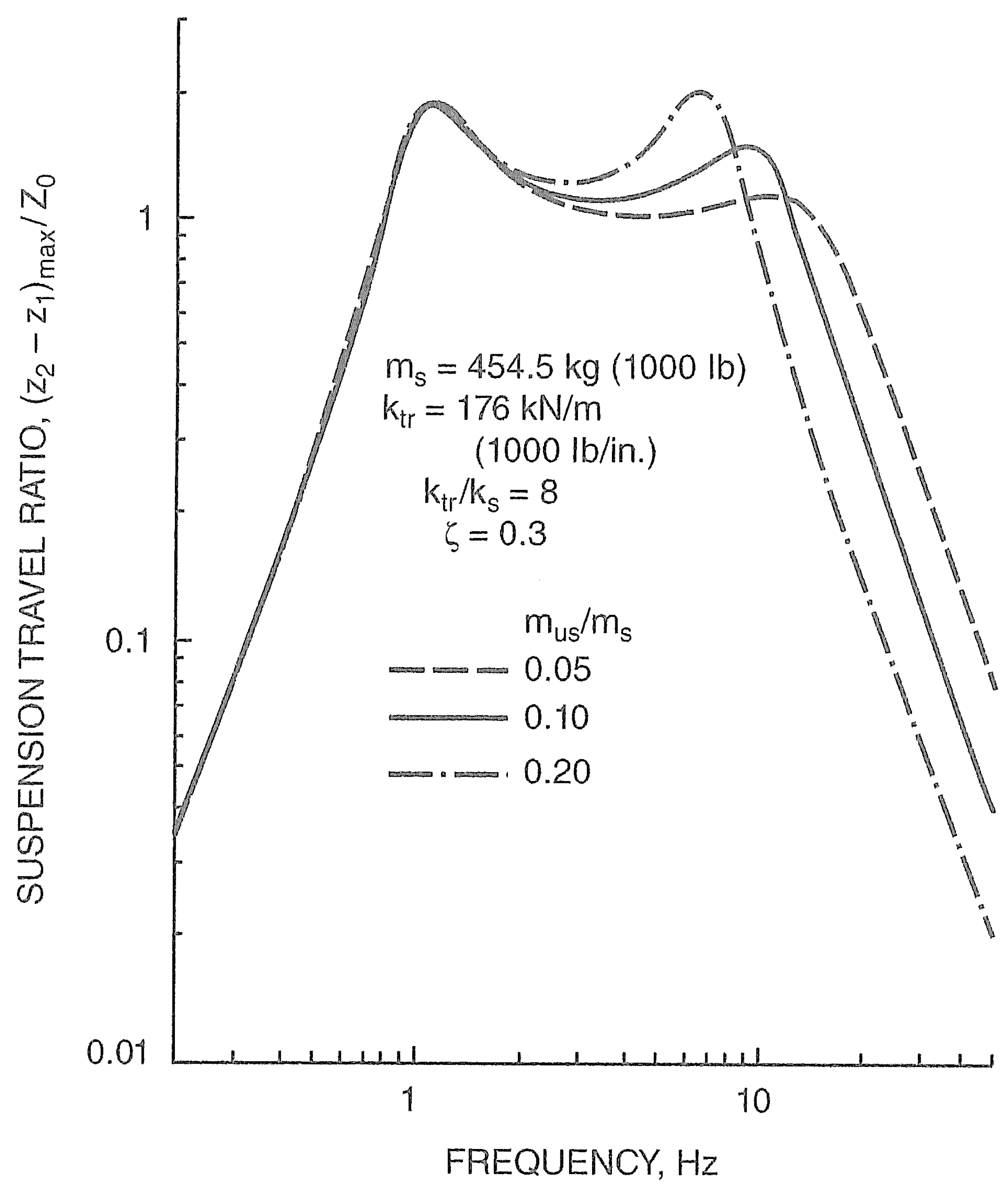


Figure 7.12

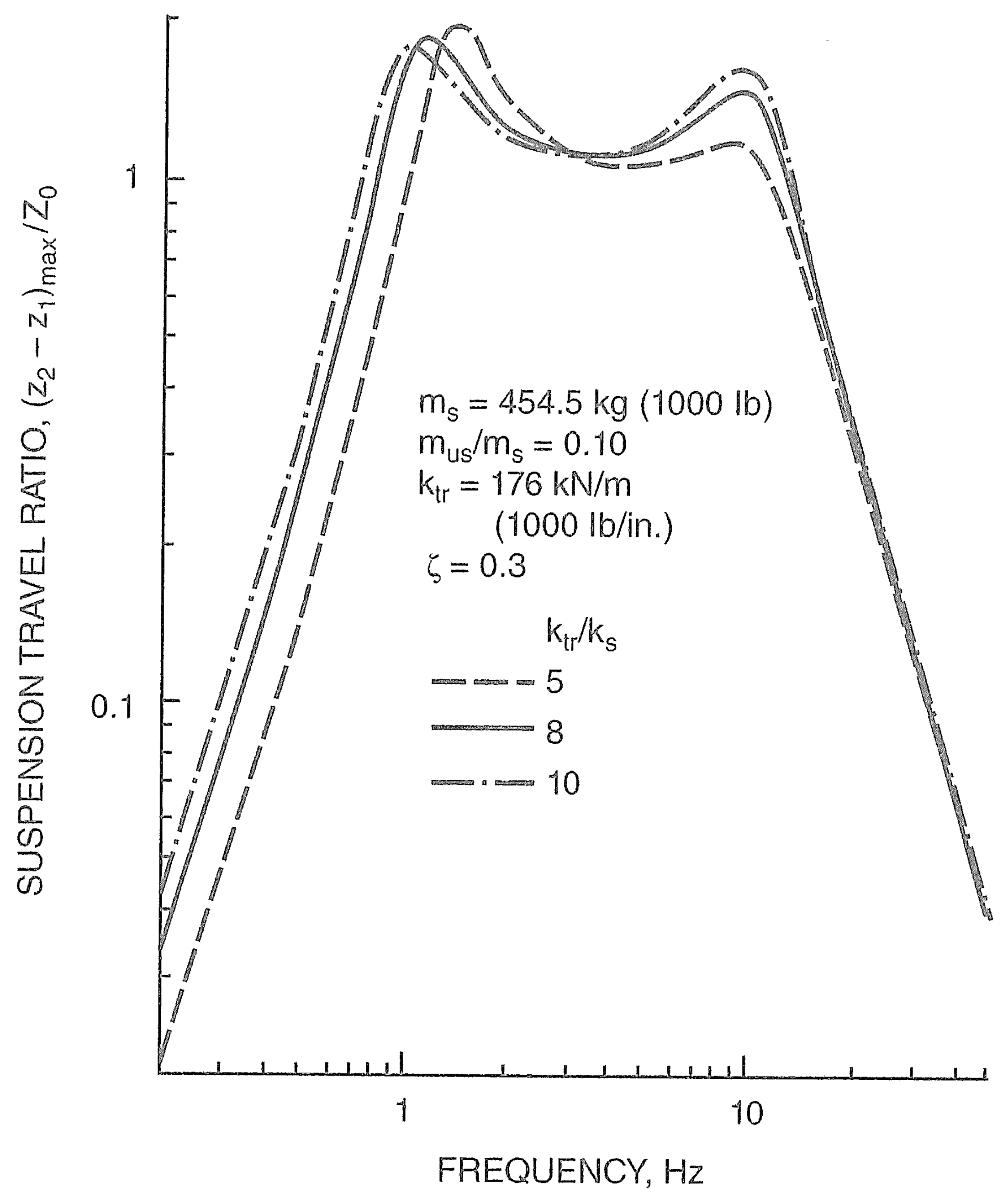


Figure 7.13

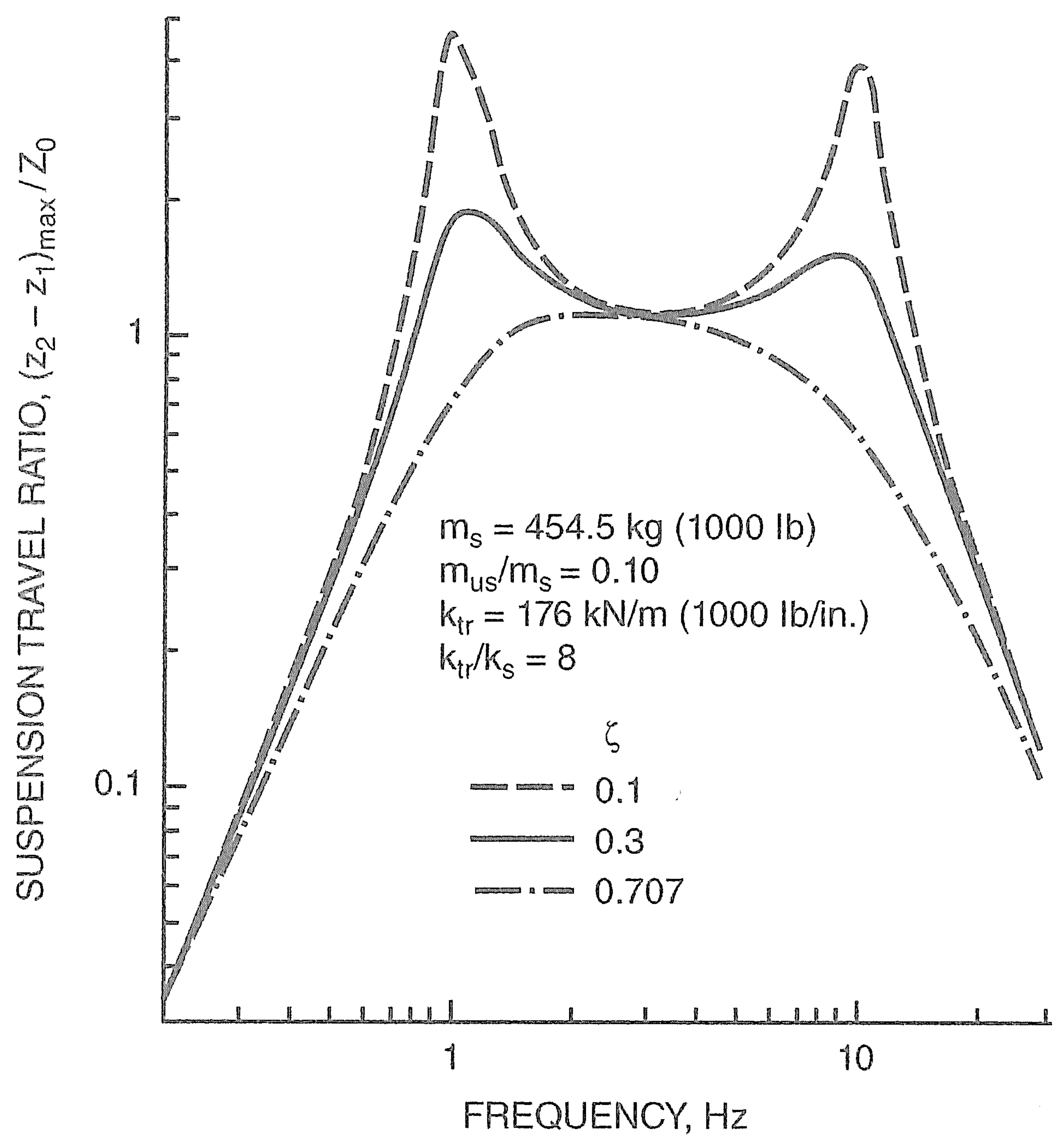


Figure 7.14

Quarter-car model: Roadholding

A measure for roadholding:

$$\frac{\max(z_0 - z_2)}{\hat{Z}_0} = |1 - G_{02}(i\omega)|$$

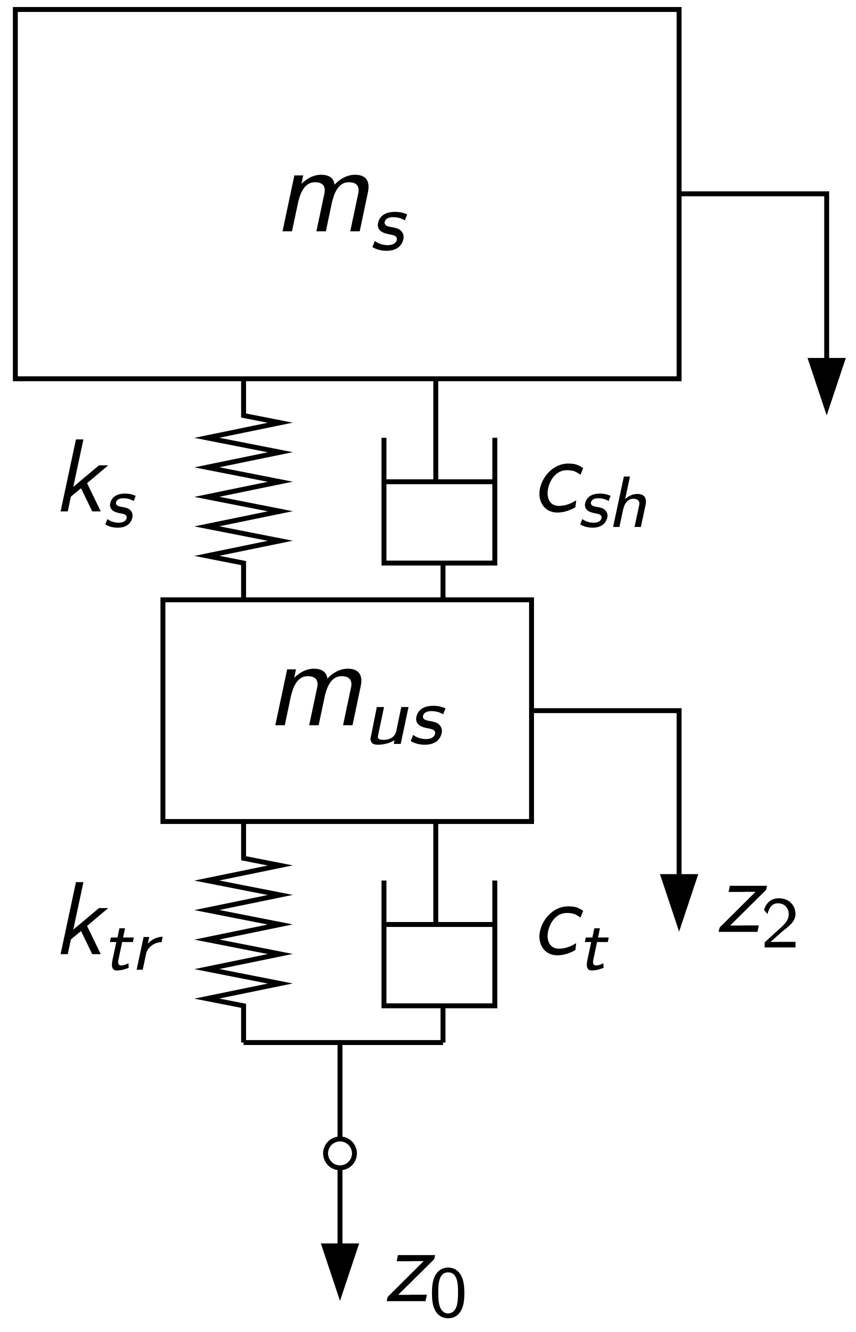


Figure 7.15—7.17 in the book show the bode plot for the transfer function from Z_0 to $Z_0 - Z_2$ for different values of the

- Unsprung mass m_{us}
- Stiffness coefficient of the suspension spring k_s
- Damping coefficient γ

Quarter-car model: Roadholding

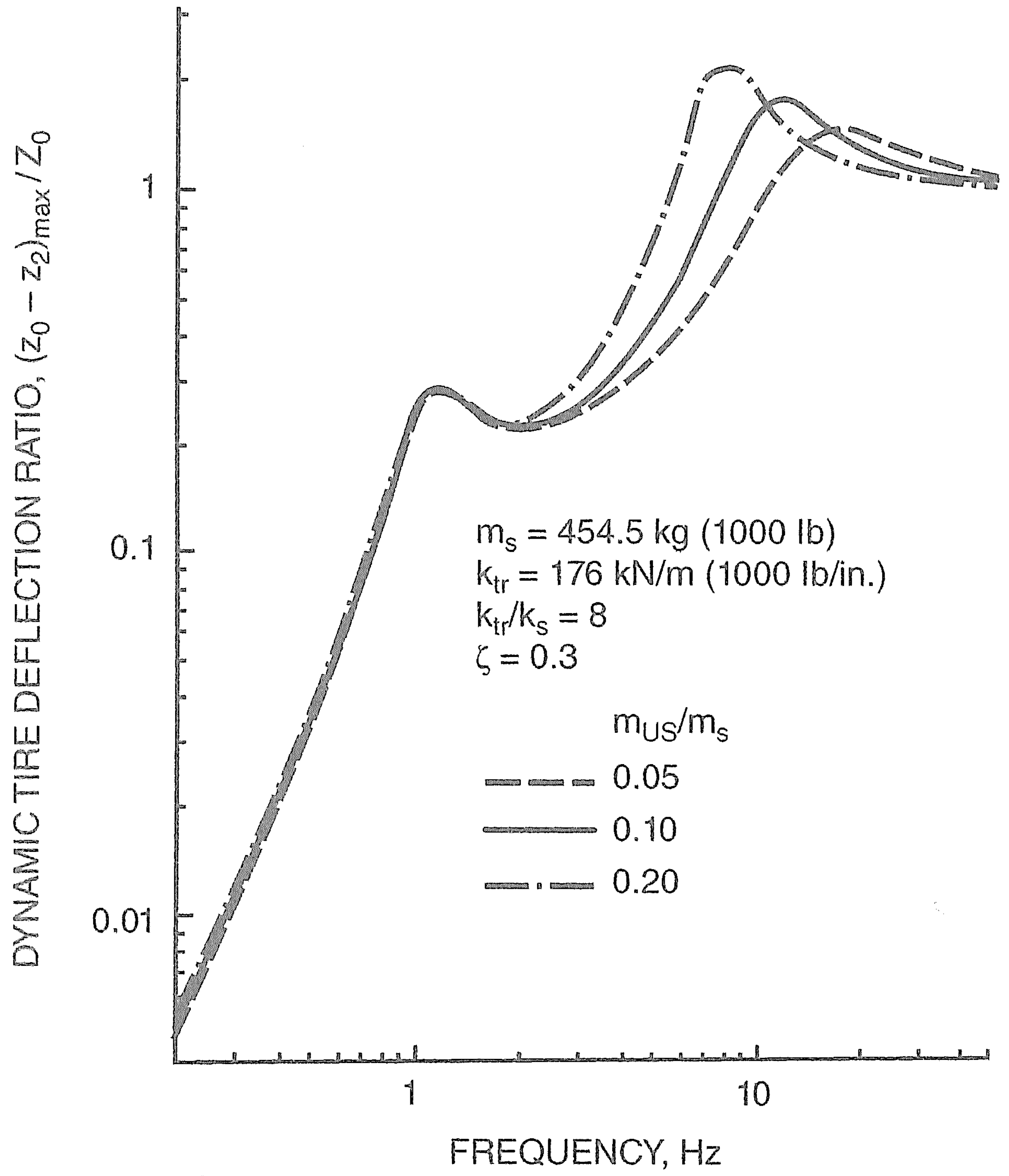


Figure 7.15

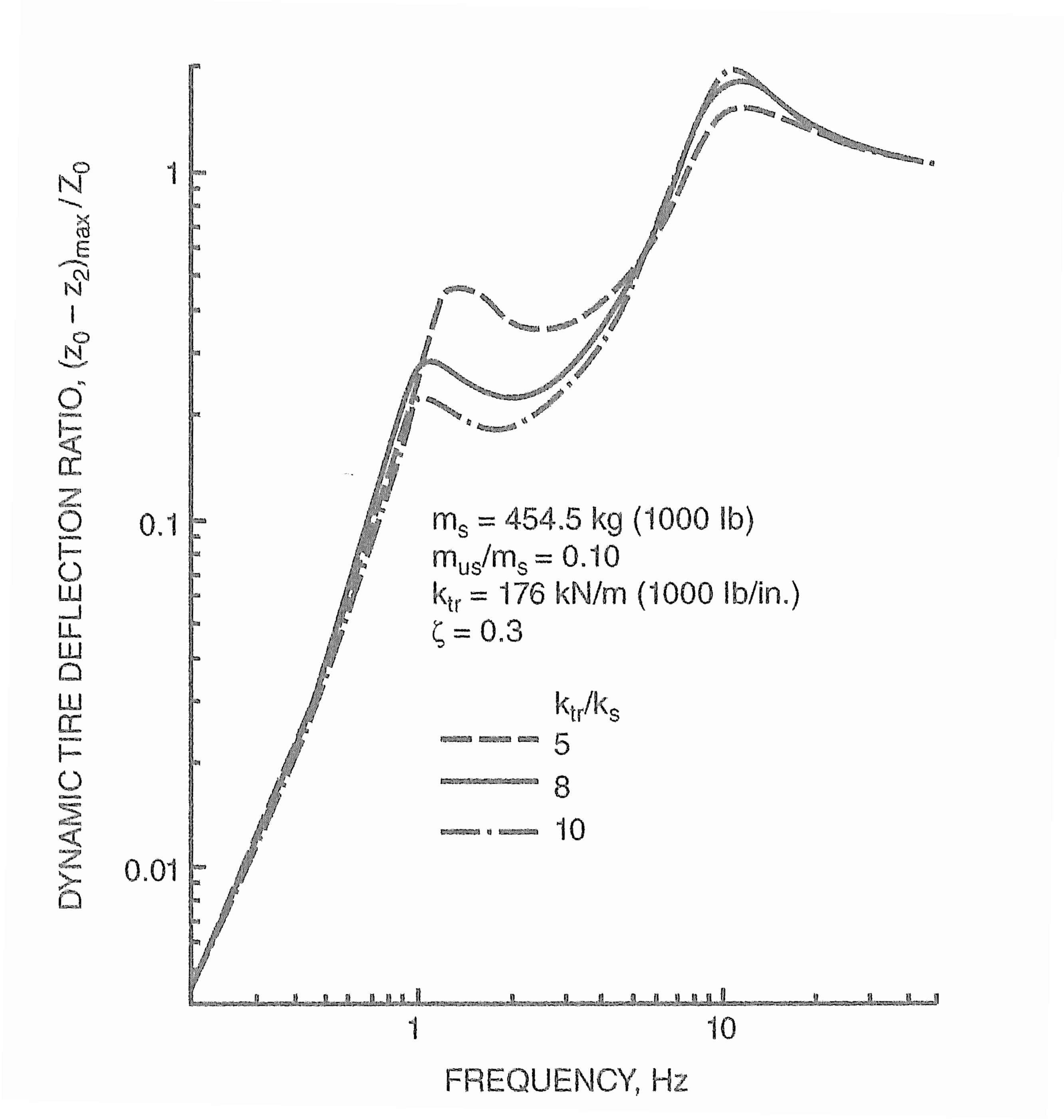


Figure 7.16

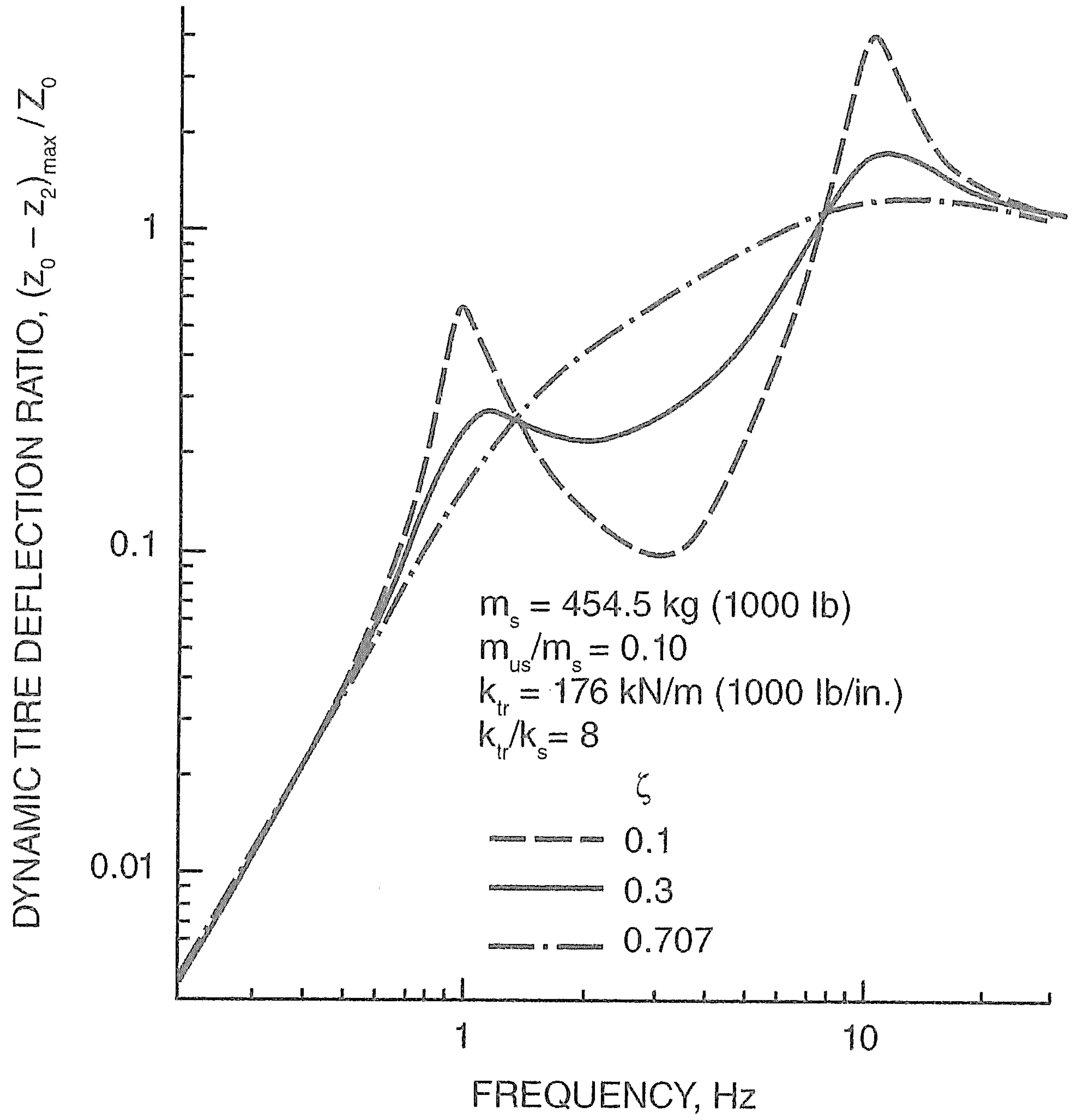


Figure 7.17

Non-linear dampers

We have consider linear dampers. An example how a non-linear damper might behave: