Vehicle Dynamics and Control

Lecture 3

LINKÖPING UNIVERSITY

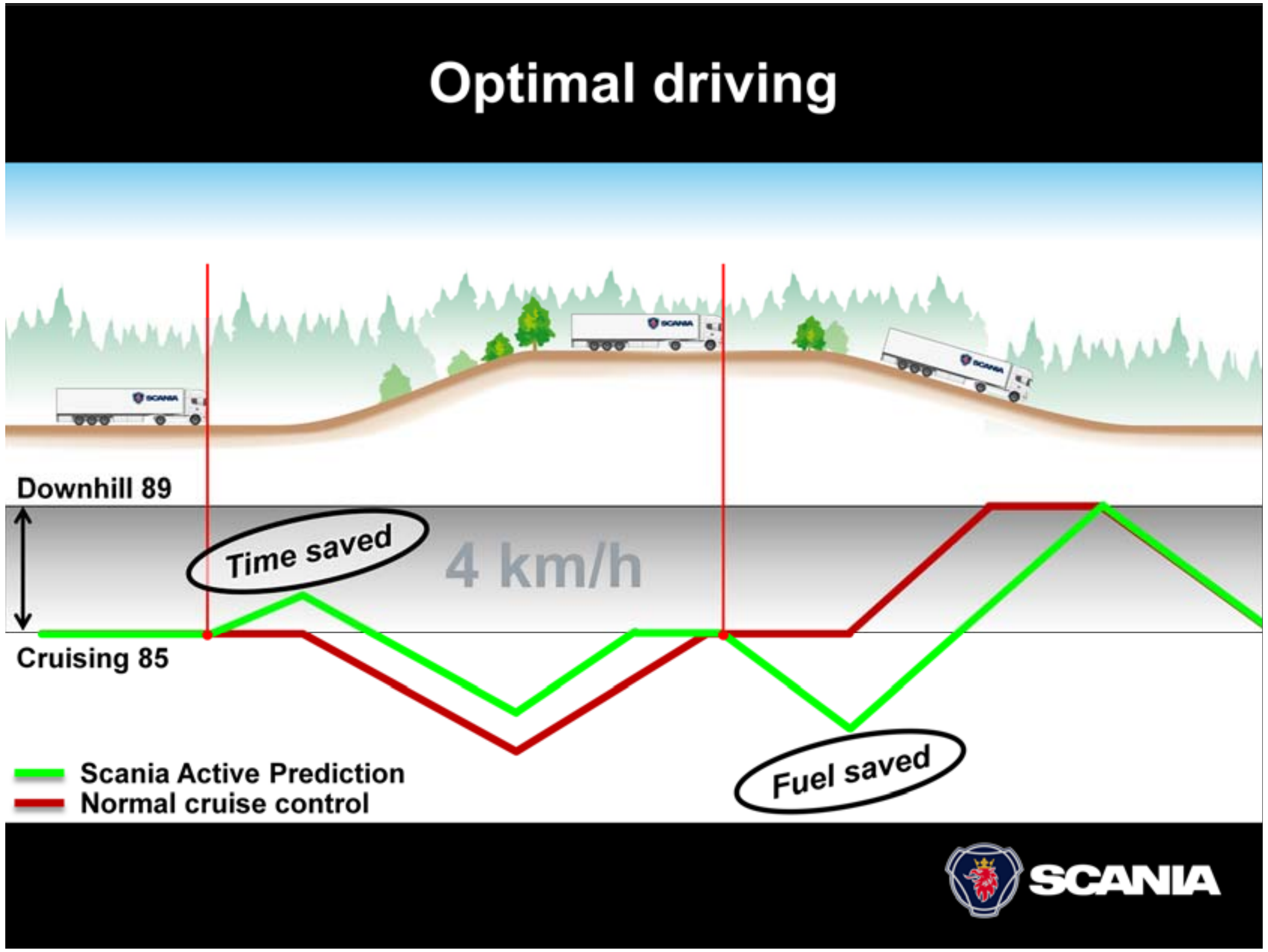
Longitudinall Dynamics: Adaptive Cruise Control

Longitudinal control

Some examples:

- CC Cruise Control
- ACC Adaptive Cruise Control
- CA Collision avoidance
- ABS Anti-Blockier-System

Longitudinal control: Cruise control

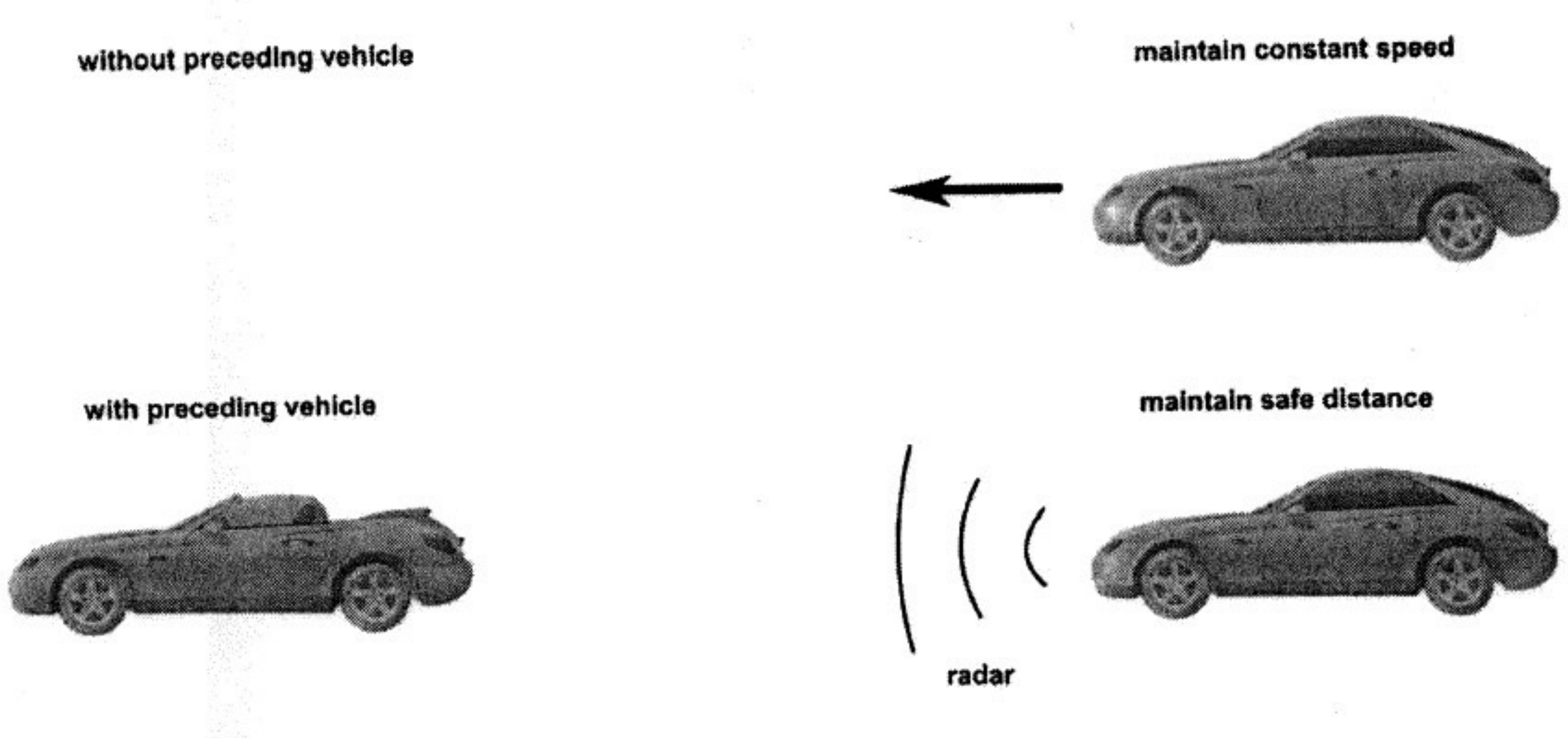


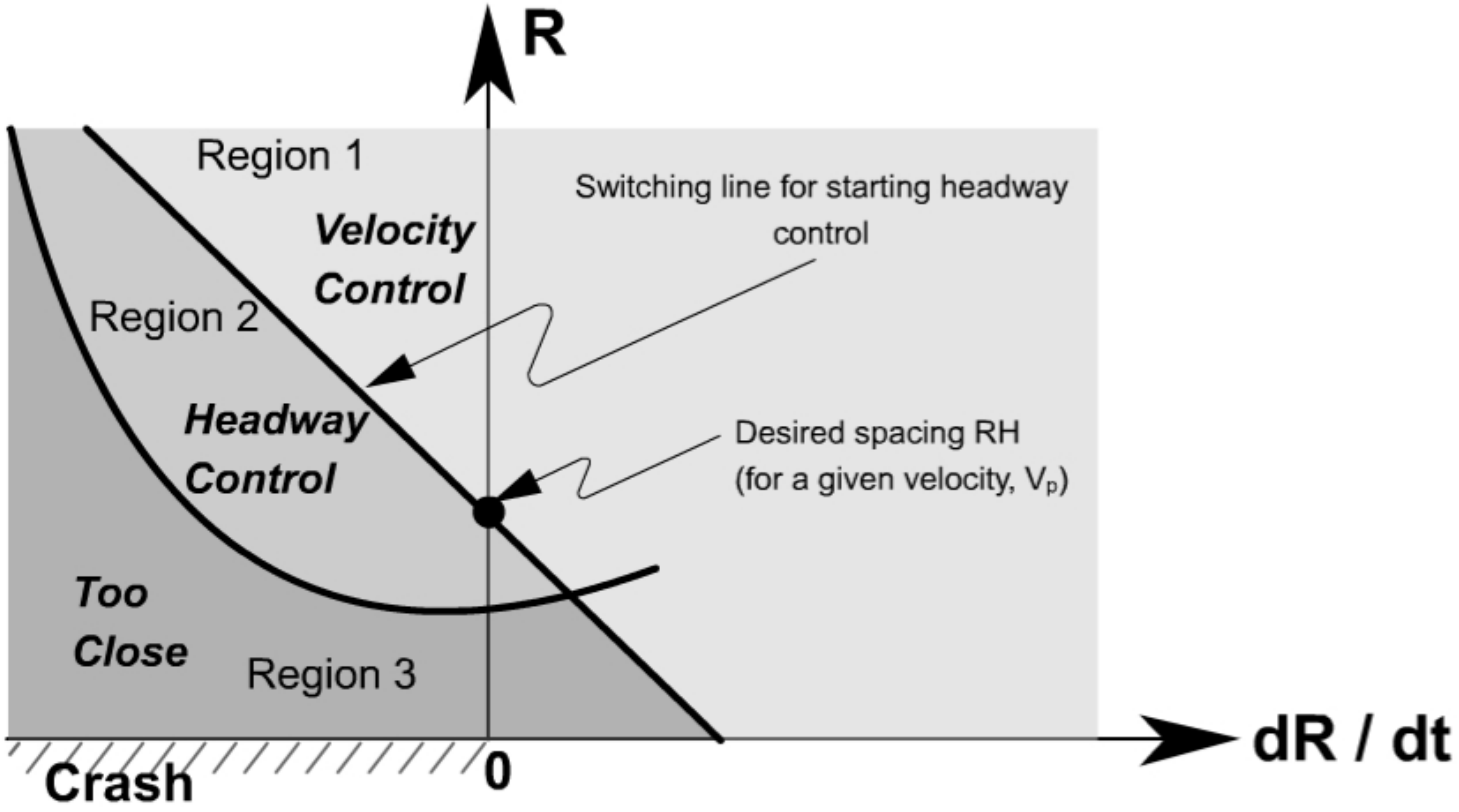
ACC Adaptive Cruise Control

Uses radar or other senors to measure the distance to other vehicles.
Control brakes and acceleration

Three different modes

- Cruise control
- Keep distance to a vehicle in front of you
- Brake to avoid collision

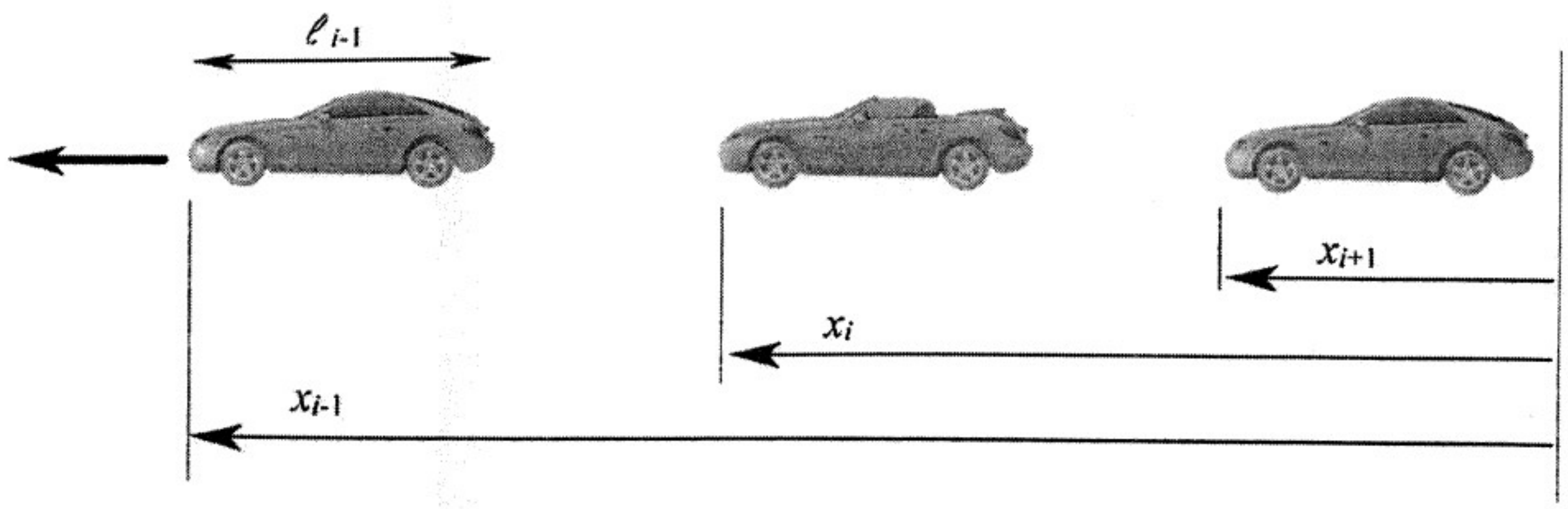


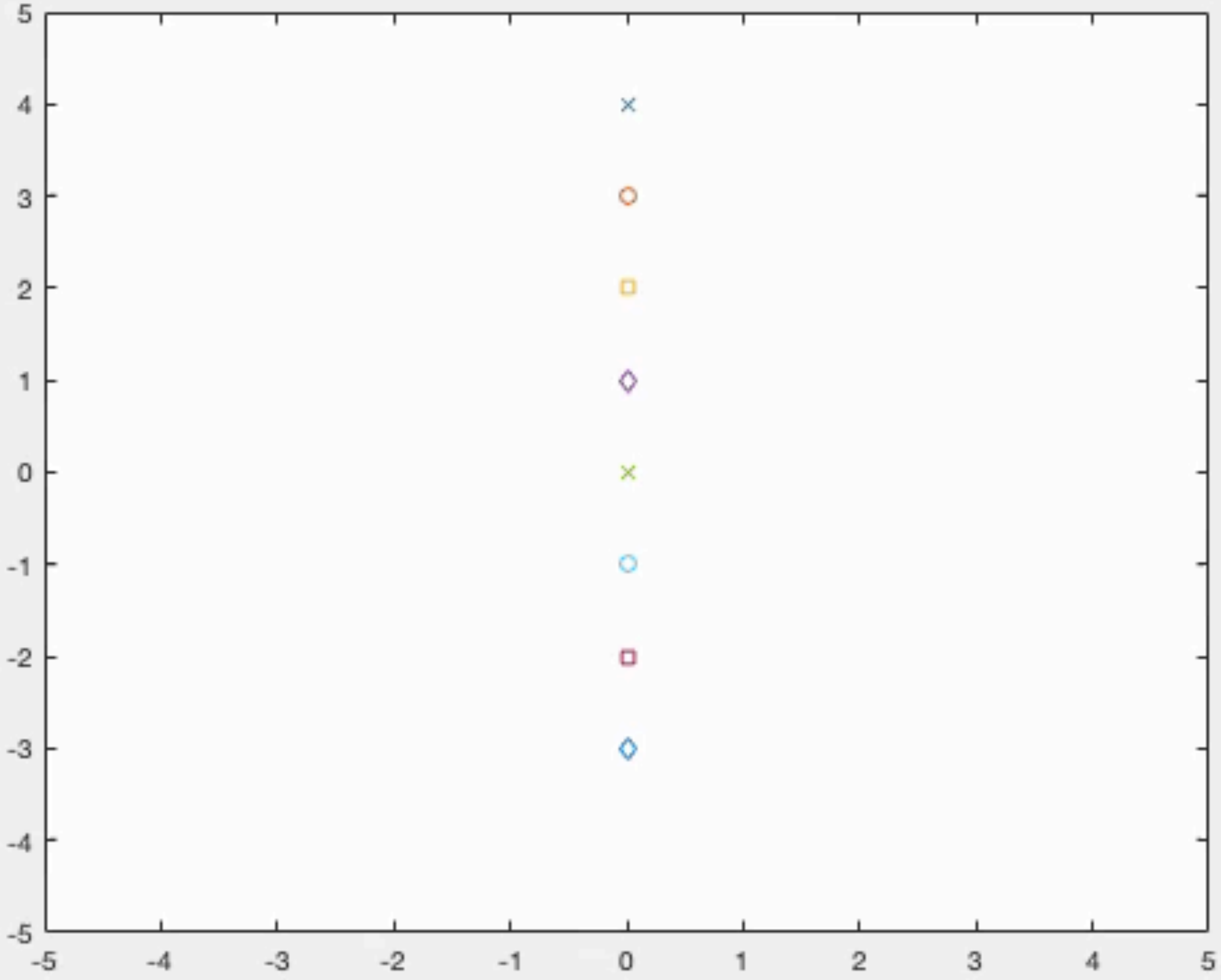


Region 1: Speed Control
Region 2: Distance Control
Region 3: Crash Control

ACC, String stability

In a long caravan with ACC in all vehicles, string stability is important





5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5

ACC, String stability

Consider a caravan where $x_i, i = 1,2,\ldots$ are the positions of the vehicles

Define

$\delta_i = x_i - x_{i-1} + L_{des}$

where L_{des} is the desired distance.

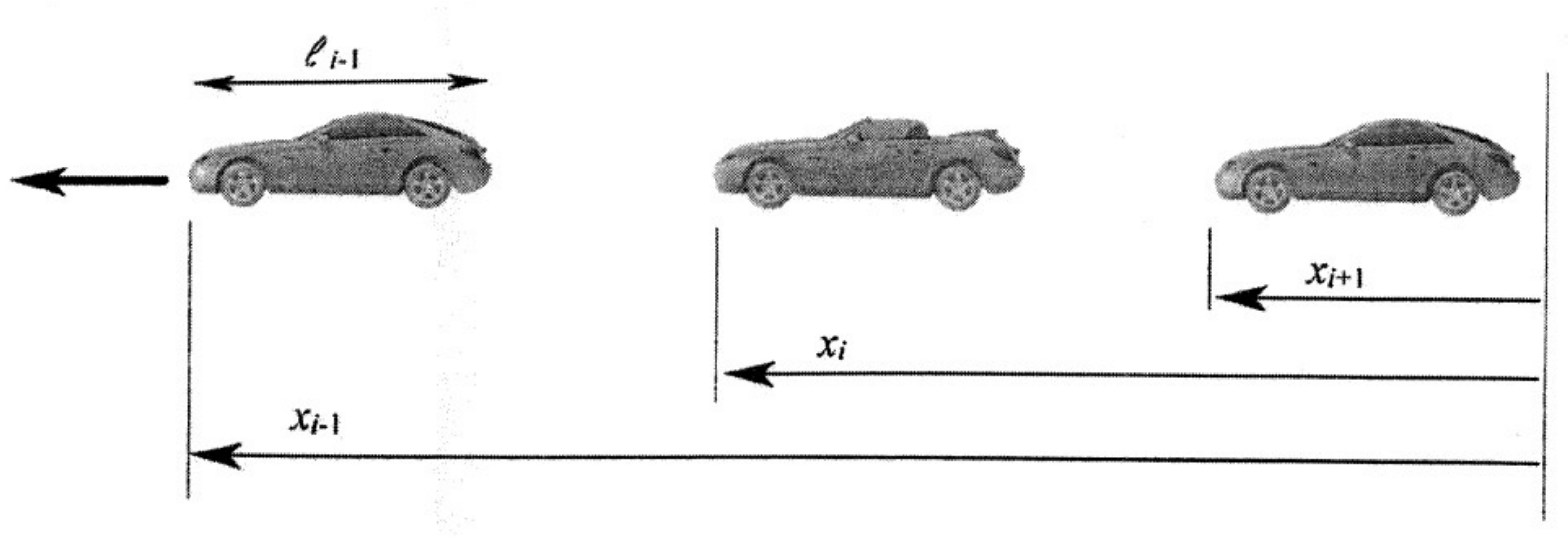
A simple longitudinal model of the vehicle

$\ddot{x}_i = u_i$

where the acceleration u_i is the control signal.

Assume that the following control strategy is used

$u_i = -k_p\delta_i - k_v\dot{\delta}_i$



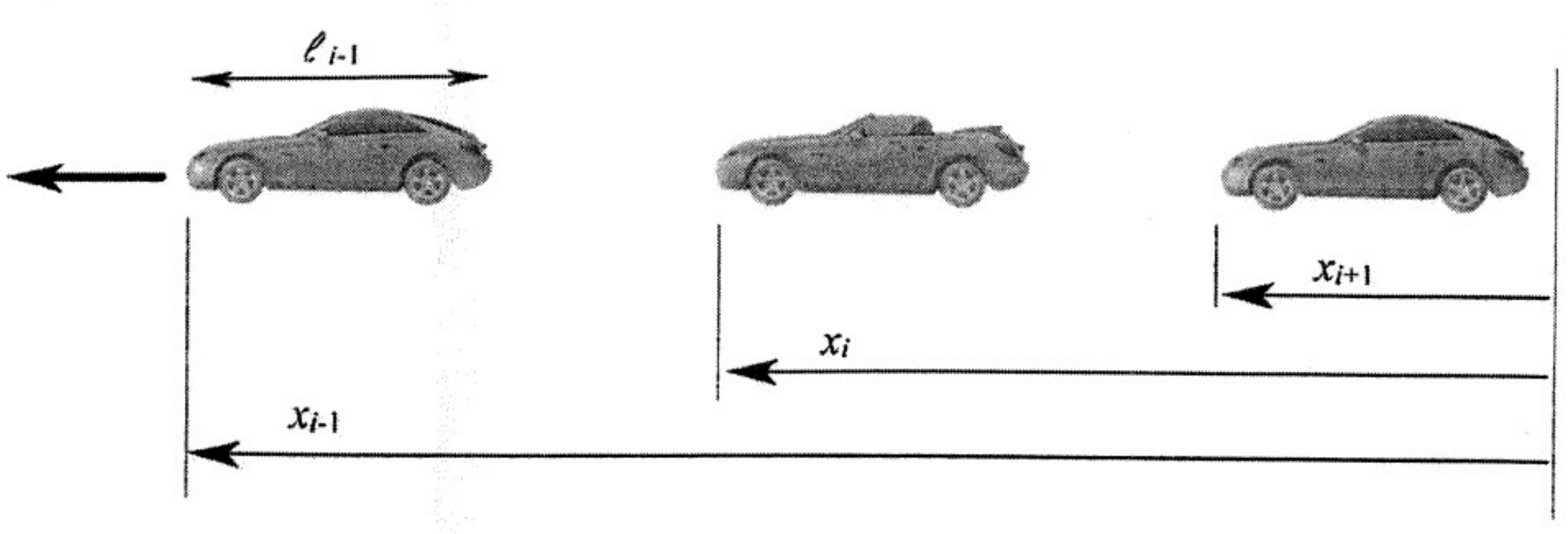
ACC, String stability

The models from the previous slide:

$$\delta_i = x_i - x_{i-1} + L_{des}$$

$$\ddot{x}_i = u_i$$

$$u_i = -k_p\delta_i - k_v\dot{\delta}_i$$



The second and third equation give:

$$s^2x_i = u_i = -sk_v\delta_i - k_p\delta_i$$

$$s^2x_{i-1} = u_{i-1} = -sk_v\delta_{i-1} - k_p\delta_{i-1}$$

and we get:

$$s^2\delta_i = s^2(x_i - x_{i-1}) = -sk_v\delta_i - k_p\delta_i + sk_v\delta_{i-1} + k_p\delta_{i-1}$$

and

$$G(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{k_v s + k_p}{s^2 + k_v s + k_p}$$

ACC, String stability

Hence, the transfer function relating the spacing errors of two consecutive vehicles is

$$G(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{k_v s + k_p}{s^2 + k_v s + k_p}$$

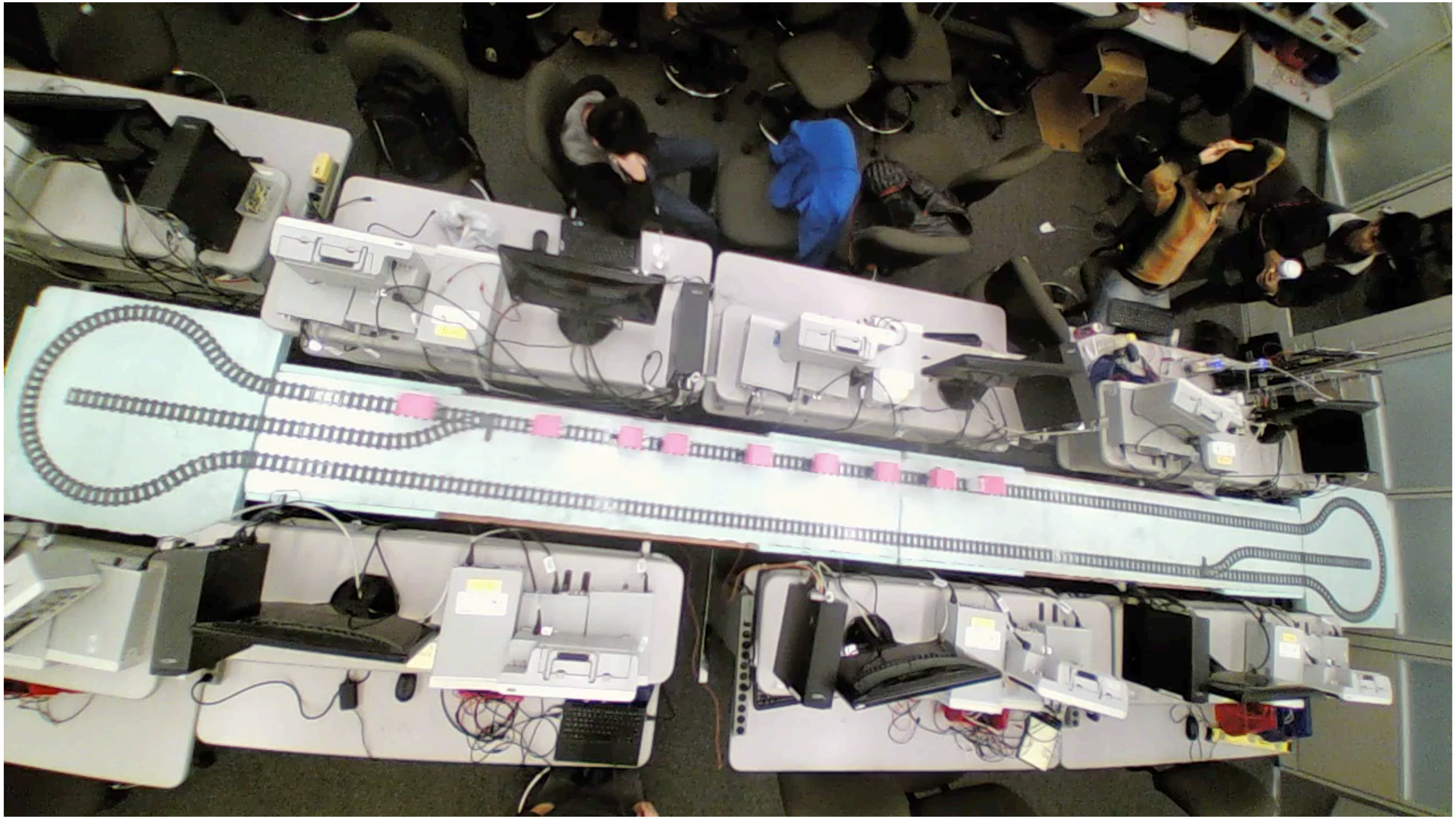
The gain is

$$|G(i\omega)| = \left| \frac{k_p + ik_v\omega}{k_p - \omega^2 + ik_v\omega} \right| = \sqrt{\frac{k_p^2 + k_v^2\omega^2}{(k_p - \omega^2)^2 + k_v^2\omega^2}}$$

and it is straightforward to show that $|G(i\omega)| > 1$ if $\omega < \sqrt{2k_p}$.

This means that the amplitude of a low frequency oscillation increases when it is transferred backwards in the caravan.

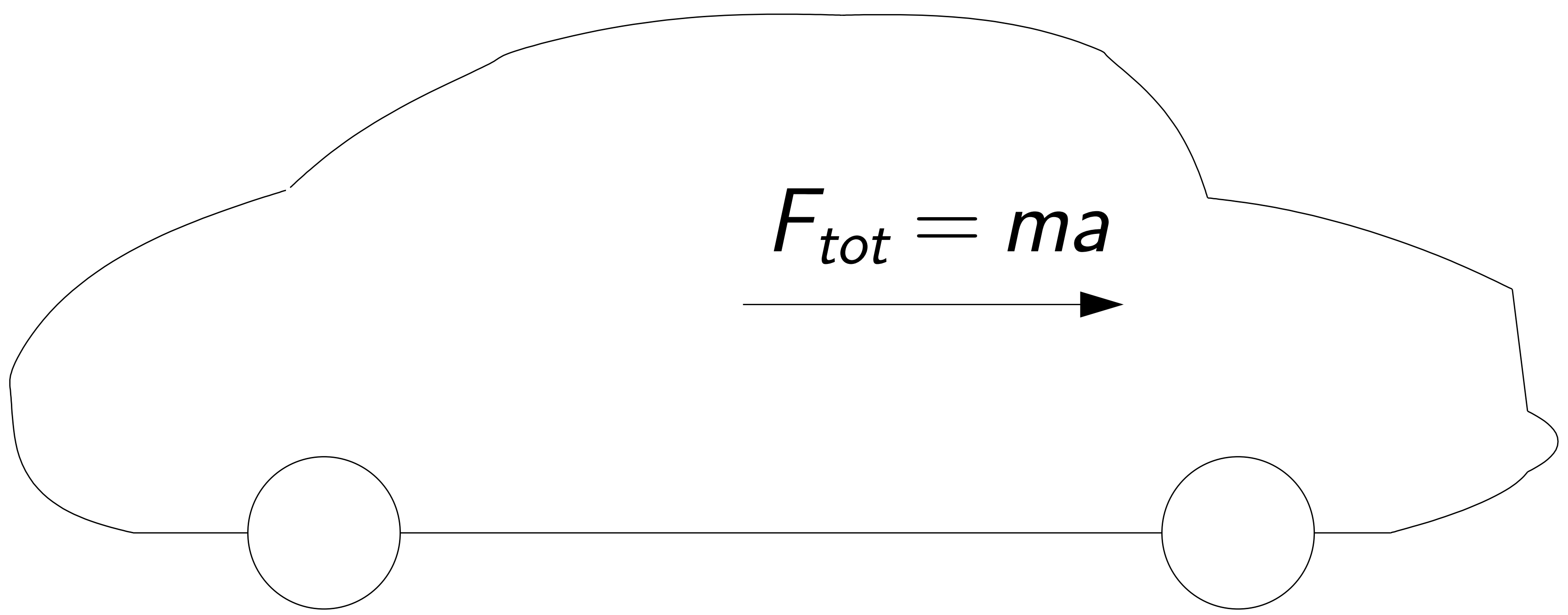
Source: *Vehicle Dynamics and Control*, Rajesh Rajamani



Longitudinal Dynamics: Stopping Distance

Longitudinal dynamics

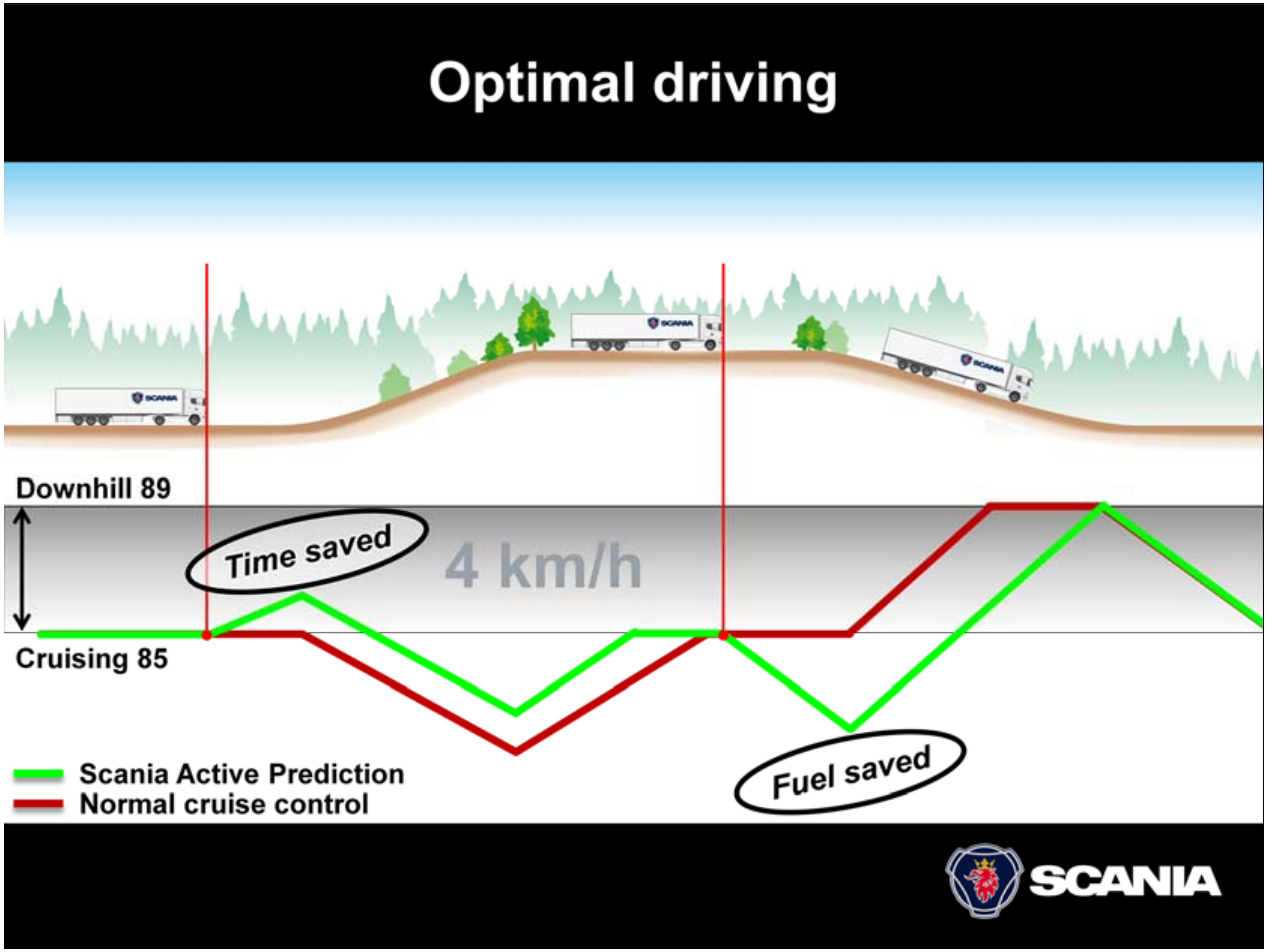
From the previous lecture:



Equation of longitudinal motion:

$$m\frac{dV}{dt} = F - R_r - R_g - R_a$$

Longitudinal control: Cruise control



Position as independent variable

In the cruise control application, the grade resistance R_g is a function of position. In this and many other cases it is natural to use position as independent variable.

The right-hand side of differential equation then becomes

$$m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx} = \frac{m}{2}\frac{d(v^2)}{dx} = \frac{d(mv^2/2)}{dx}$$

and we obtain

$$\frac{m}{2}\frac{d(v^2)}{dx} = F - R_r - R_g - R_a$$

Note that the previously introduced models for R_r and R_a are linear functions of v^2.

Kinetic energy

It follows that

$$d(mv^2/2) = (F - R_r - R_a)\,dx - mg\,dh$$

where

- $d(mv^2/2)$: change of kinetic energy.
- $(F - R_r - R_a)\,dx$: work
- $mg\,dh = mg\sin\theta_s\,dx$: change of potential energy

Stopping distance: First case

Given an initial speed V, the objective is to determine the stopping distance S.

From the previous slides:

$$\frac{m}{2}\frac{d(v^2)}{dx} = -F_b - R_r - R_g - R_a$$

Before the general case is analyzed, some special cases will be considered.

First case: Neglect all forces except F_b. In this case we get:

$$\frac{m}{2}d(v^2) = -F_b\,dx$$

Calculate the integrals from start to stop

$$\int_{V^2}^{0} \frac{m}{2}\,d(v^2) = -\int_{0}^{S} F_b\,dx$$

Note how the intervals of the integrals were chosen!

Stopping distance: First case

The results is

$$\frac{mV^2}{2} = F_b S$$

i.e.

Initial kinetic energy $=$ Stopping distance $\times$ Brake force

and

$$S = \frac{mV^2}{2F_b}$$

Stopping distance: Second case

The **second case** includes the grade resistance $mg\sin\theta_s$:

$$\int_{V^2}^{0}\frac{m}{2}\,d(v^2) = -\int_{0}^{S}(F_b + mg\sin\theta_s)\,dx$$

and the result is

$$\frac{mV^2}{2} = F_b S + mg\sin\theta_s S$$

i.e.

Initial kinetic energy $=$ Stopping distance $\times$ Brake force
$+$Change in potential energy

and

$$S = \frac{mV^2/2}{F_b + mg\sin\theta_s}$$

Stopping distance: General case

In a more **general case**, we have the differential equation

$$\frac{m}{2}\frac{d(v^2)}{dx} = -F_b - mg\sin\theta_s - f_r mg\cos\theta_s - C_{ae}v^2$$

It is a separable differential equation

$$\frac{m}{2}\int_{V^2}^{0}\frac{d(v^2)}{F_b + mg\sin\theta_s + f_r mg\cos\theta_s + C_{ae}v^2} = -\int_0^S dx$$

and the result is

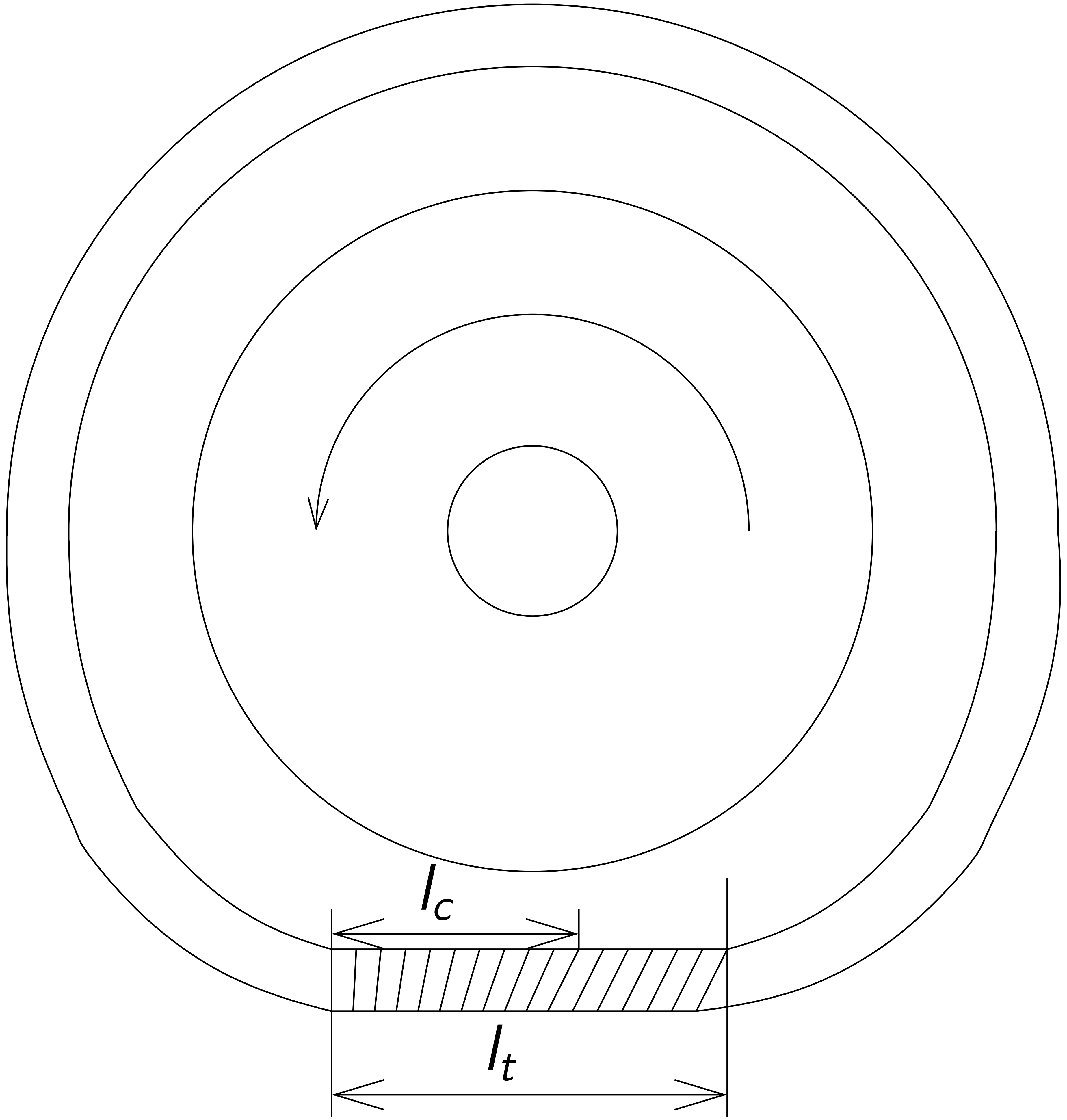
$$S = \frac{m}{2C_{ae}}\log\left(\frac{F_b + mg\sin\theta_s + f_r mg\cos\theta_s + C_{ae}V^2}{F_b + mg\sin\theta_s + f_r mg\cos\theta_s}\right)$$

$$= \frac{m}{2C_{ae}}\log\left(1 + \frac{C_{ae}V^2}{F_b + mg\sin\theta_s + f_r mg\cos\theta_s}\right)$$

Tyre modelling: The Brush Model, cont'd

Brush model

From Lecture 1

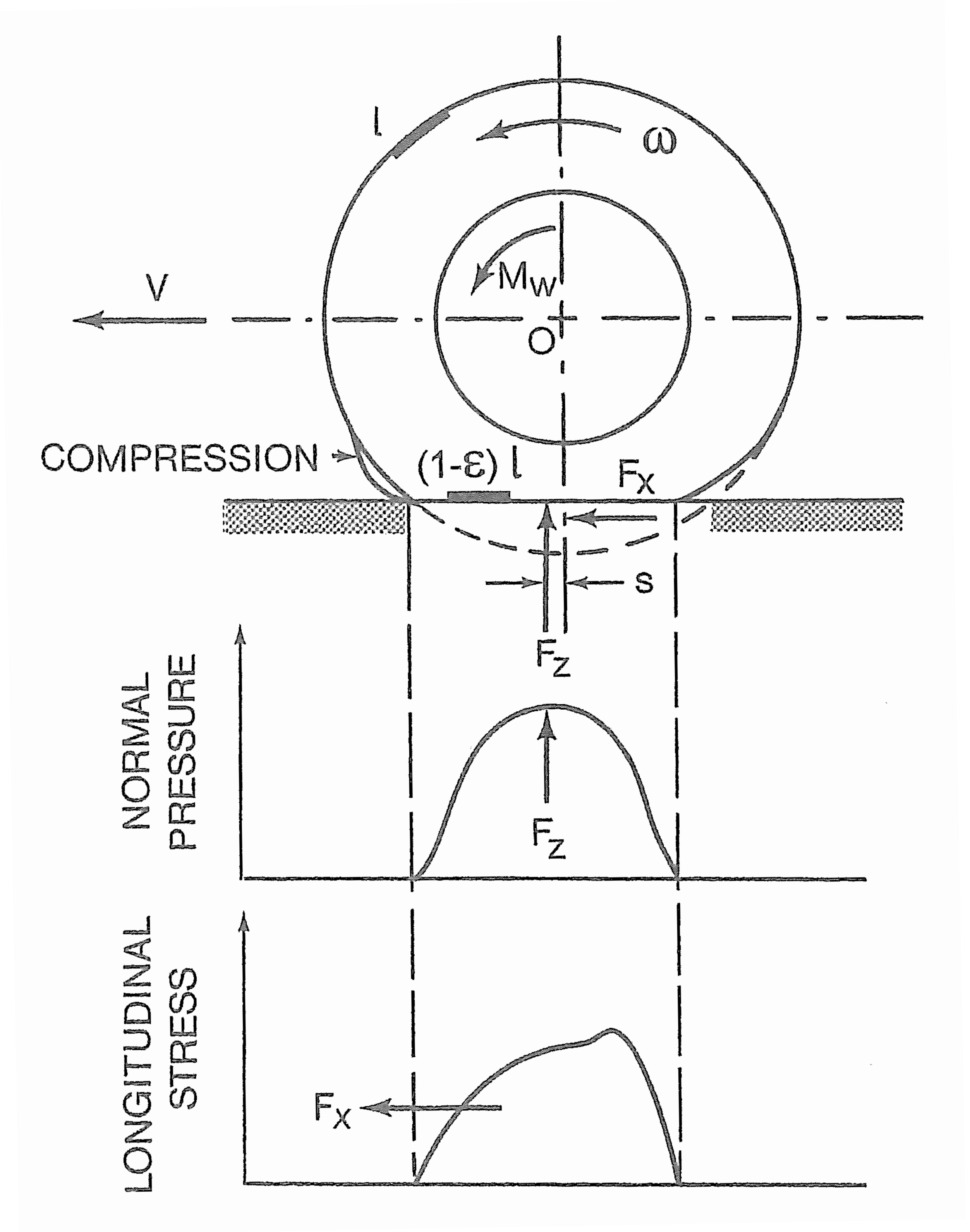


Brush model: Normal pressure

It was assumed that the normal pressure was constant in the contact region.

According to Figure 1.15 a parabola shaped distribution seem more reasonable, i.e.,

$$\frac{dF_z}{dx} = Cx(l_t - x)$$



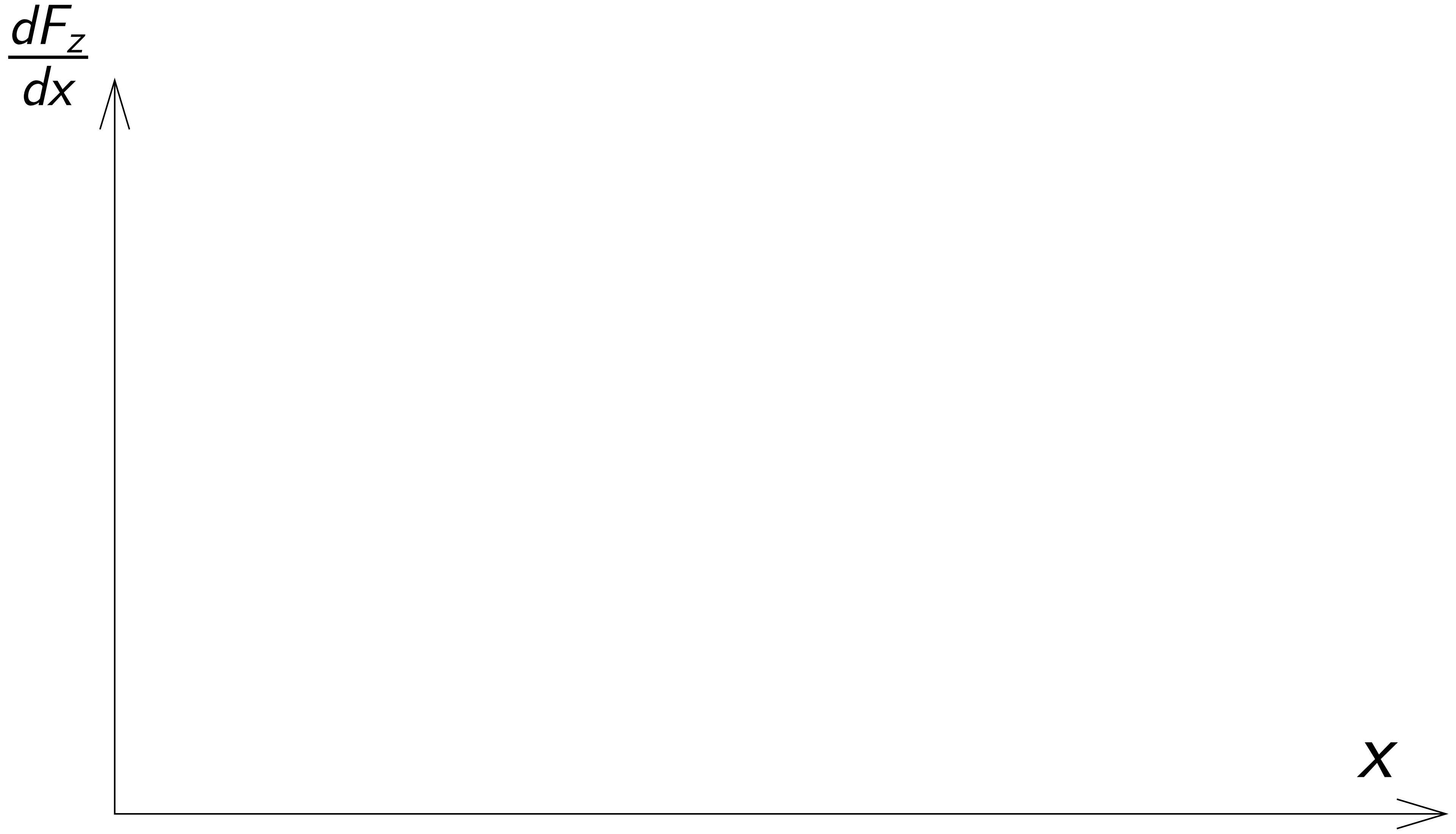
In the adhesion region we assume

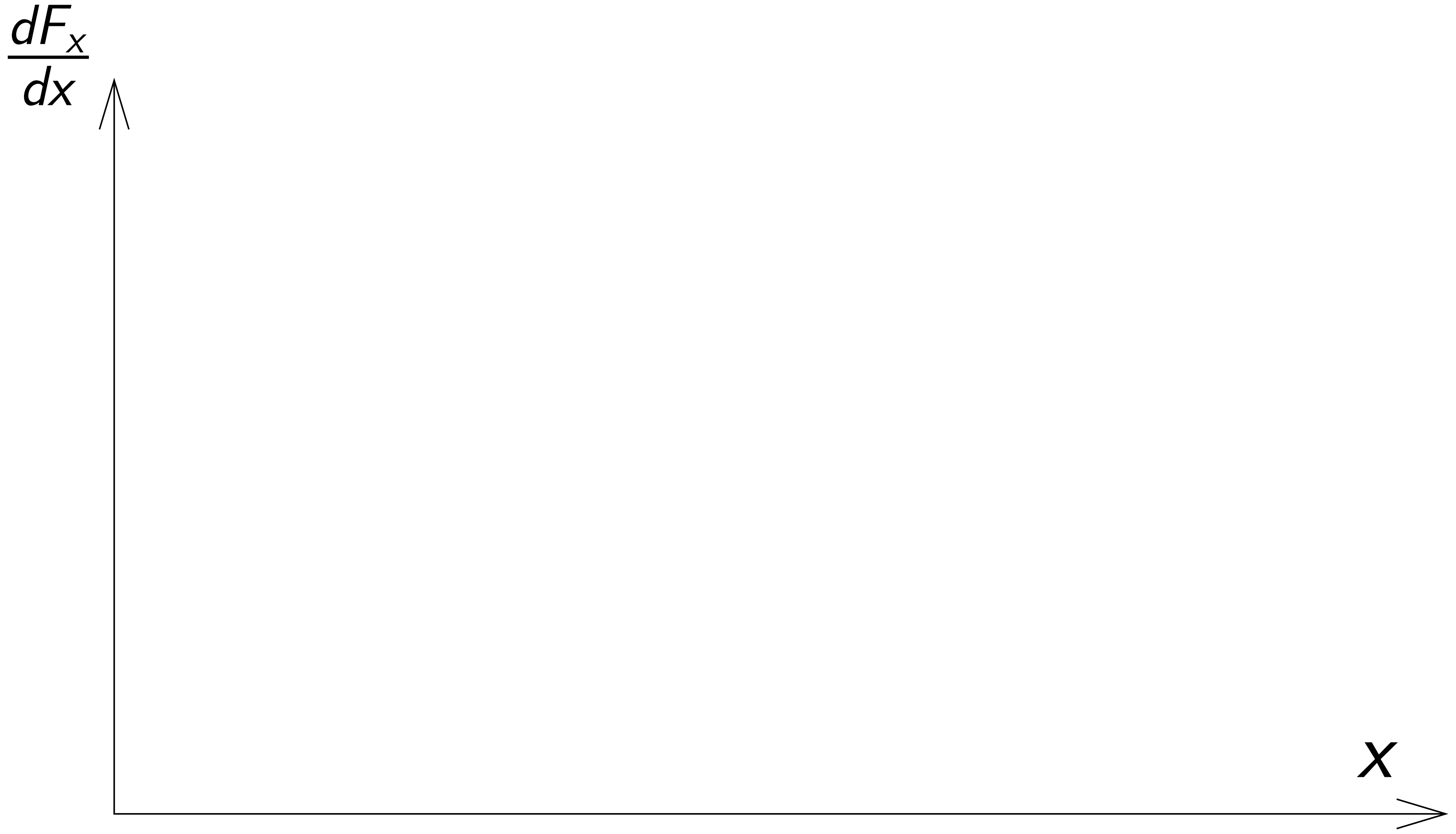
$$\frac{dF_x}{dx} = k_t \cdot i \cdot x \text{ , and } \frac{dF_x}{dx} < \mu_p \frac{dF_z}{dx},$$

as before.

Brush model: Normal pressure

Sketch the normal and longitudinal force distributions:





The longitudinal force F_x is the area under the curve to the right.

Brush model: Sliding friction

Figure 1.16 shows the longitudinal force as a function of slip:

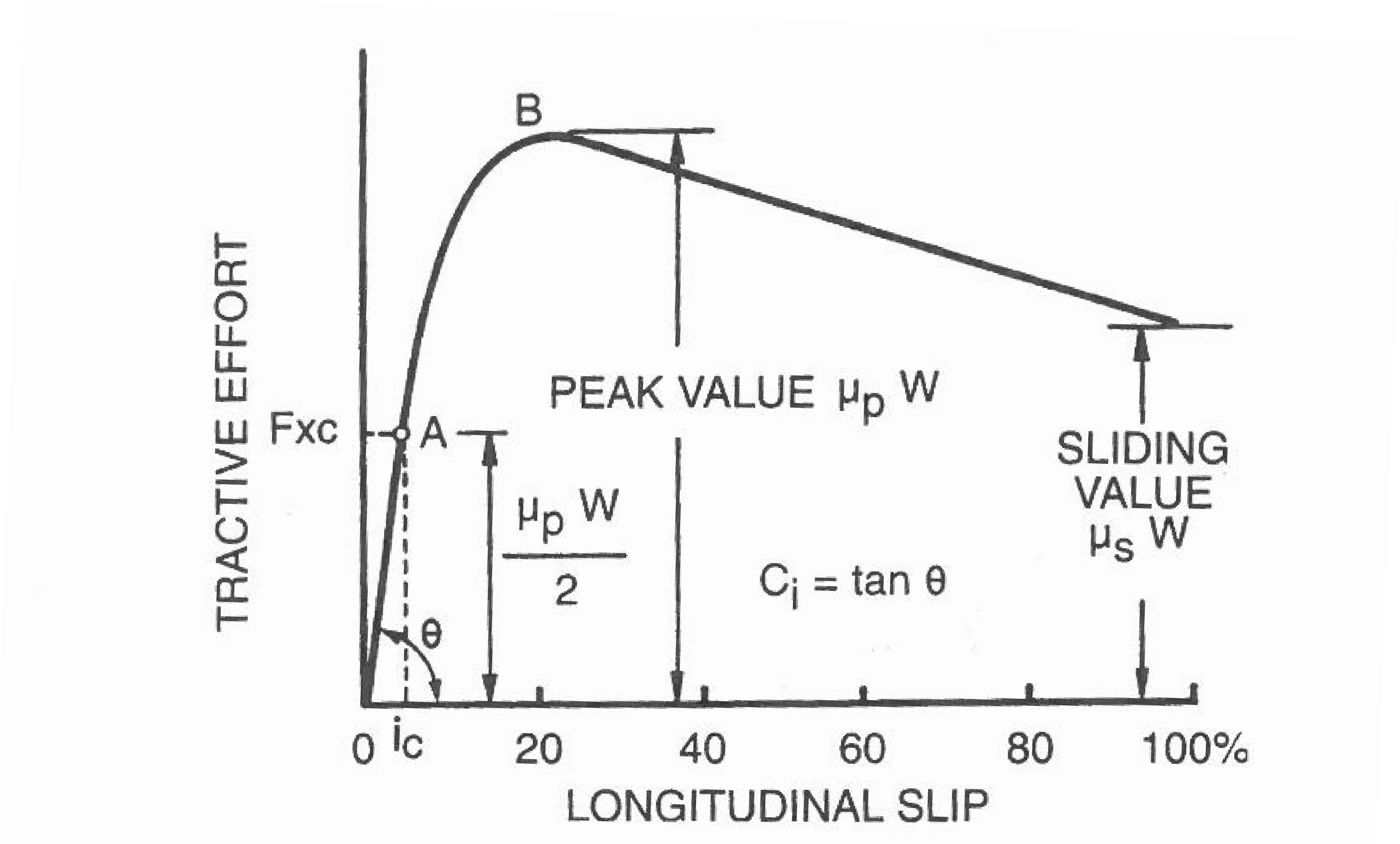


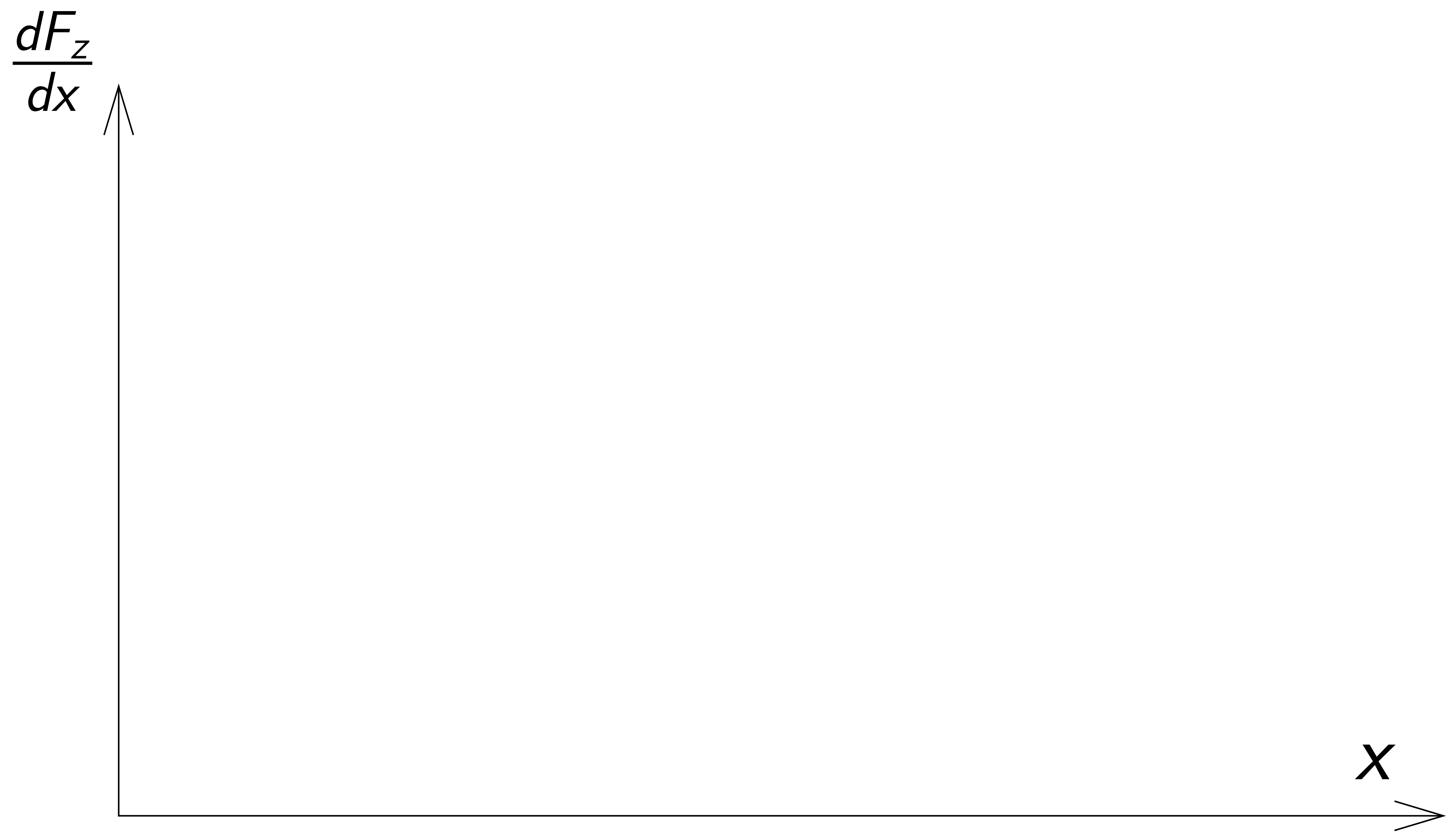
Fig. 1.16 Variation of tractive effort with longitudinal slip of a tire.

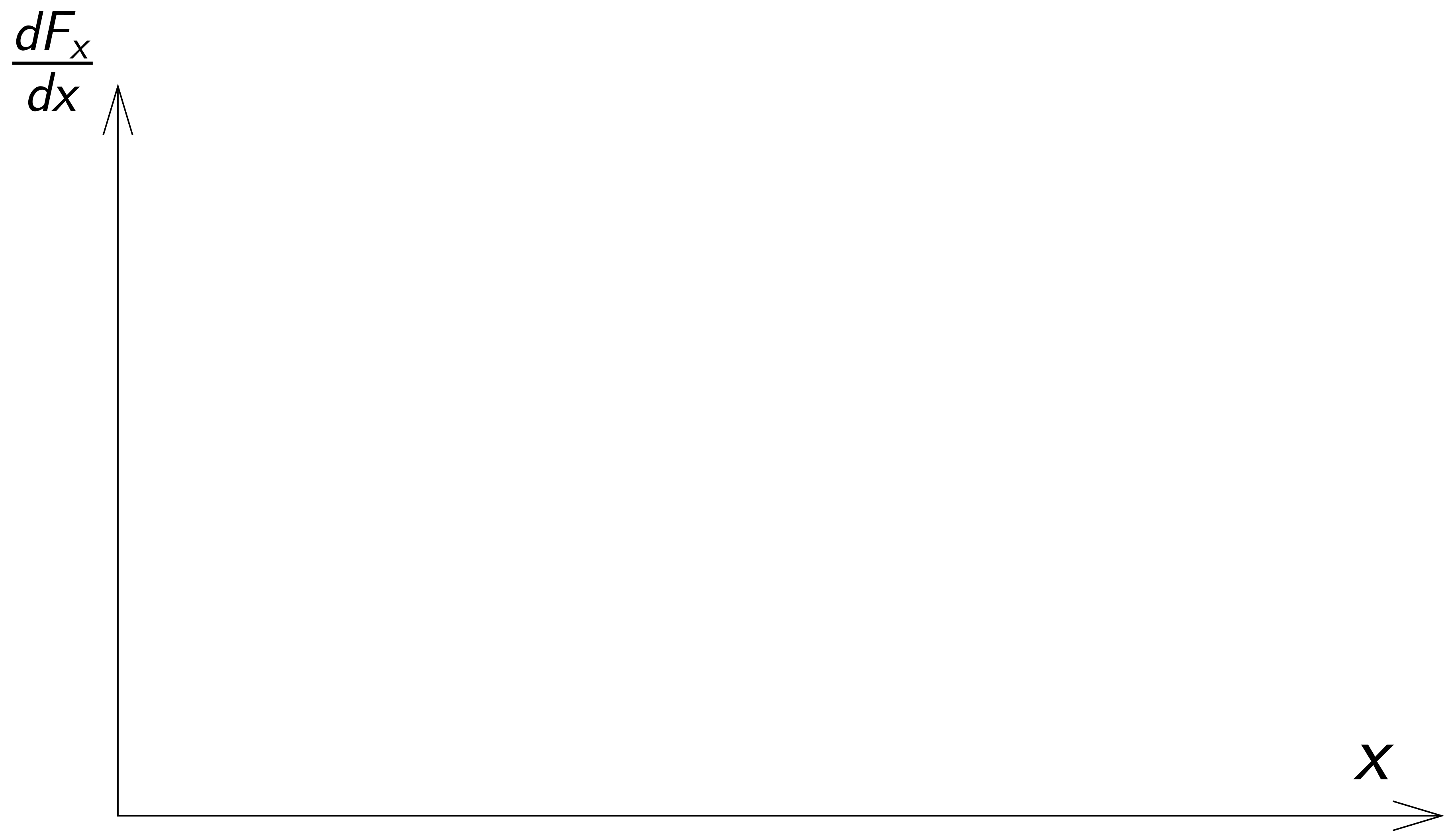
It can be seen that the force reaches a peak value and then decreases.

Assume that the sliding friction μ_s is lower than the friction μ_p in the adhesive re
and that the normal force distribution is constant in the contact region.

Brush model: Sliding friction

Sketch the normal and longitudinal force distributions:





The longitudinal force F_x is the area under the curve to the right.

Application: Estimations of Coefficient of Friction μ

Estimation of coefficient of friction μ

The coefficient of friction has been an important part when analyzing the braking and acceleration performance. Now, one example will be presented on how to estimate the coefficient.

The approach is based on the approximation

$$\frac{F_x}{W} = K(\mu) \cdot i$$

where it is assumed that the gradient K is a function of μ.
If we first find an estimate of K, then we can calculate μ.

To be able to calculate K, we will first estimate

- Longitudinal force F_x
- Normal force W
- Longitudinal slip i

Friction: Longitudinal force F_x

Assume that longitudinal acceleration is measured.

Longitudinal model

$$ma = F_x - R_a - R_r - R_g$$

The longitudinal force is now given by

$$F_x = m(a + g\sin\theta_s) + R_a + R_r$$

where

m is estimated mass

$a + g\sin\theta_s$ is measured by the accelerometer

R_a and R_r are calculated using empirical models

Friction: Normal force W

Normal force

$$W_f = \frac{l_2}{L} mg - \frac{h}{L}(R_a + m(a + g \sin \theta_s))$$

$$W_r = \frac{l_1}{L} mg + \frac{h}{L}(R_a + m(a + g \sin \theta_s))$$

Friction: Longitudinal slip i

Assume that the car is front-wheel driven. Then there is no slip at the rear-wheel:

$$i_r = 1 - \frac{V_x}{\omega_r r_r} = 0$$

Modern cars have sensors measuring angular speed with high precision, since this information is needed by the ABS-system.

The sensors at the rear wheels can therefore be used to calculate V_x and then the slip at the front wheel can be calculated

$$i_f = 1 - \frac{V_x}{\omega_f r_f}$$

Now, F_{xf}, W_f, and i_f at the front wheels are known and it possible to estimate K and μ using

$$\frac{F_x}{W} = K(\mu) \cdot i$$

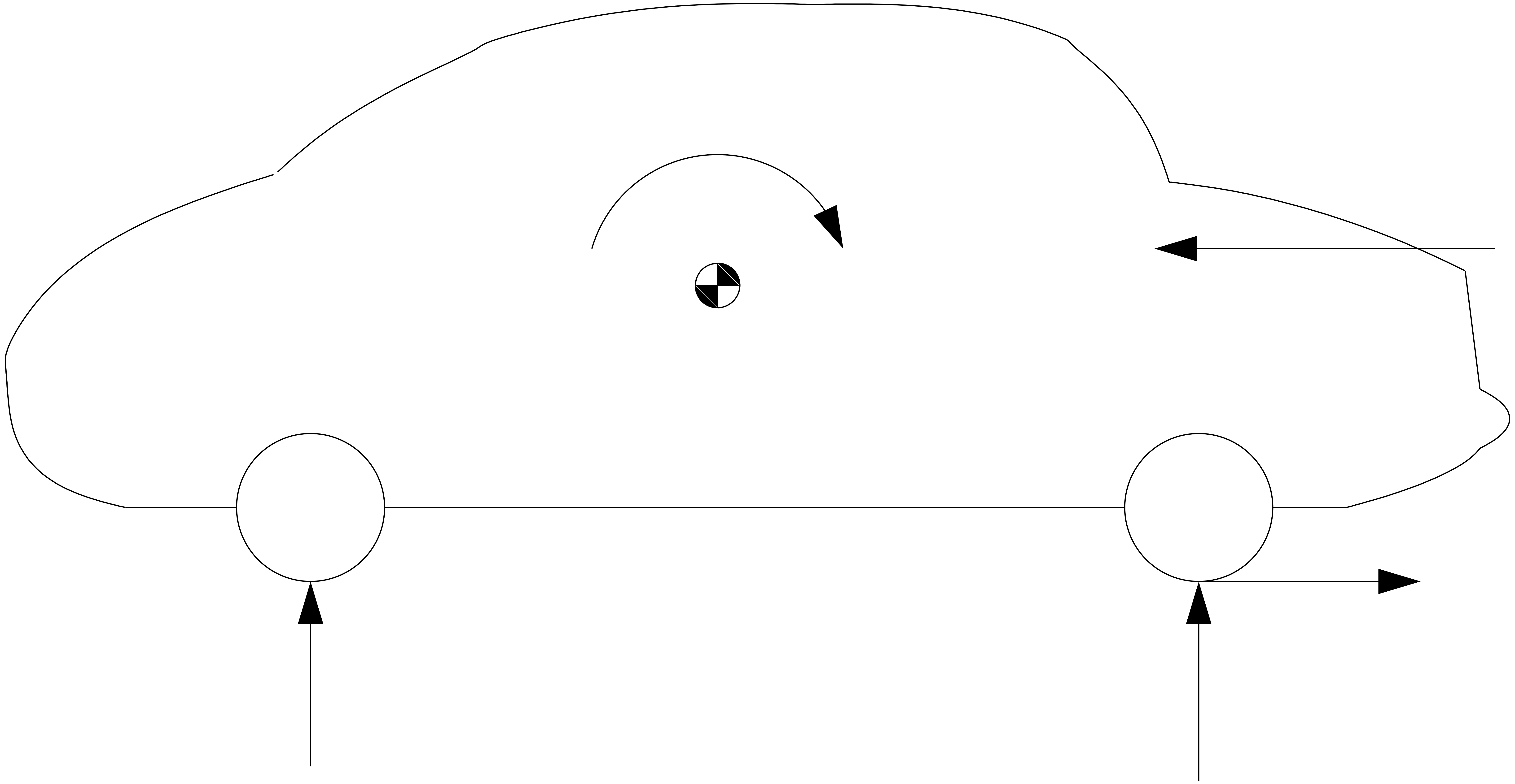
Longitudinal Control: Brake Force Distribution, cont'd

Brake force distribution

From lecture 2

$$\frac{K_{bf}}{K_{br}} = \frac{l_2 + h\mu}{l_1 - h\mu}$$

$$K_{bf} + K_{br} = 1$$



Electronic Brake-force Distribution

$$W_f = \frac{1}{L}(Wl_2 + h(F_b + F_r))$$

och

$$W_r = \frac{1}{L}(Wl_1 - h(F_b + F_r))$$

If we neglect aerodynamic resistance we get

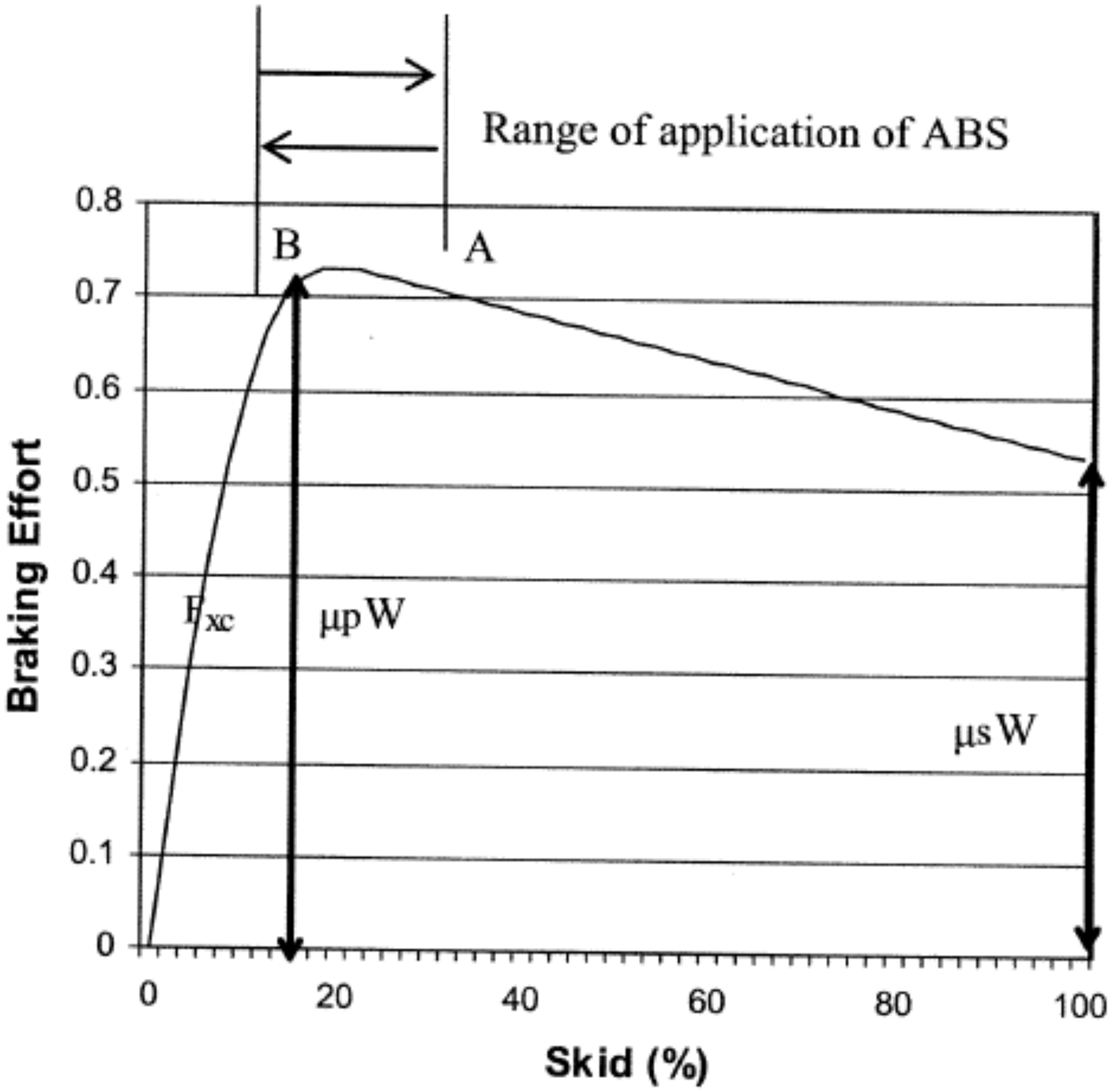
$$F_b + F_r = m(a + g\sin\theta)$$

Assume that we measure the longitudinal acceleration $a + g\sin\theta$

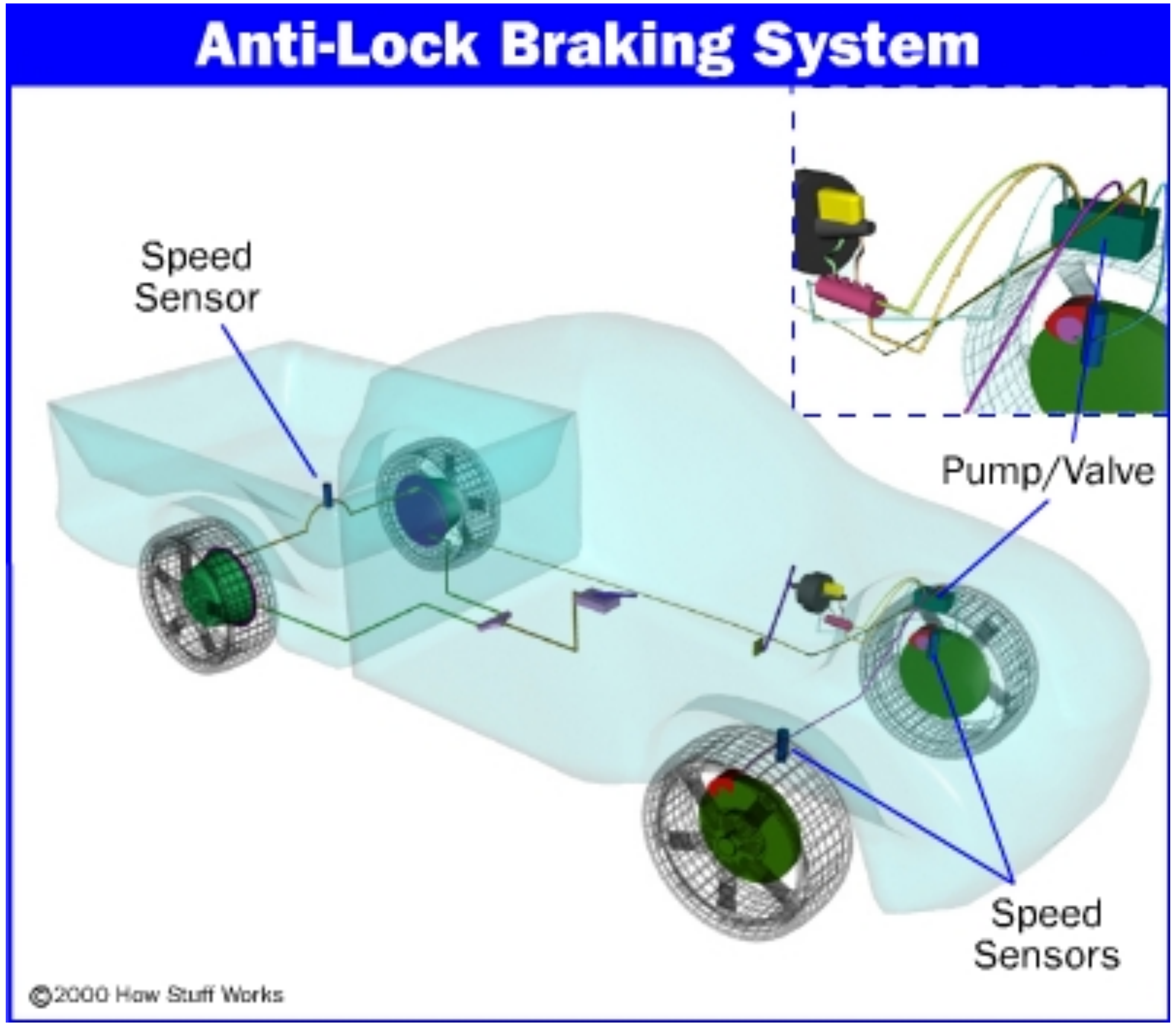
Now, we can use this information to distribute the brake forces so that all wheels start sliding at the same time, without knowing μ?

Longitudinal Control: Antilock Braking Systems (ABS)

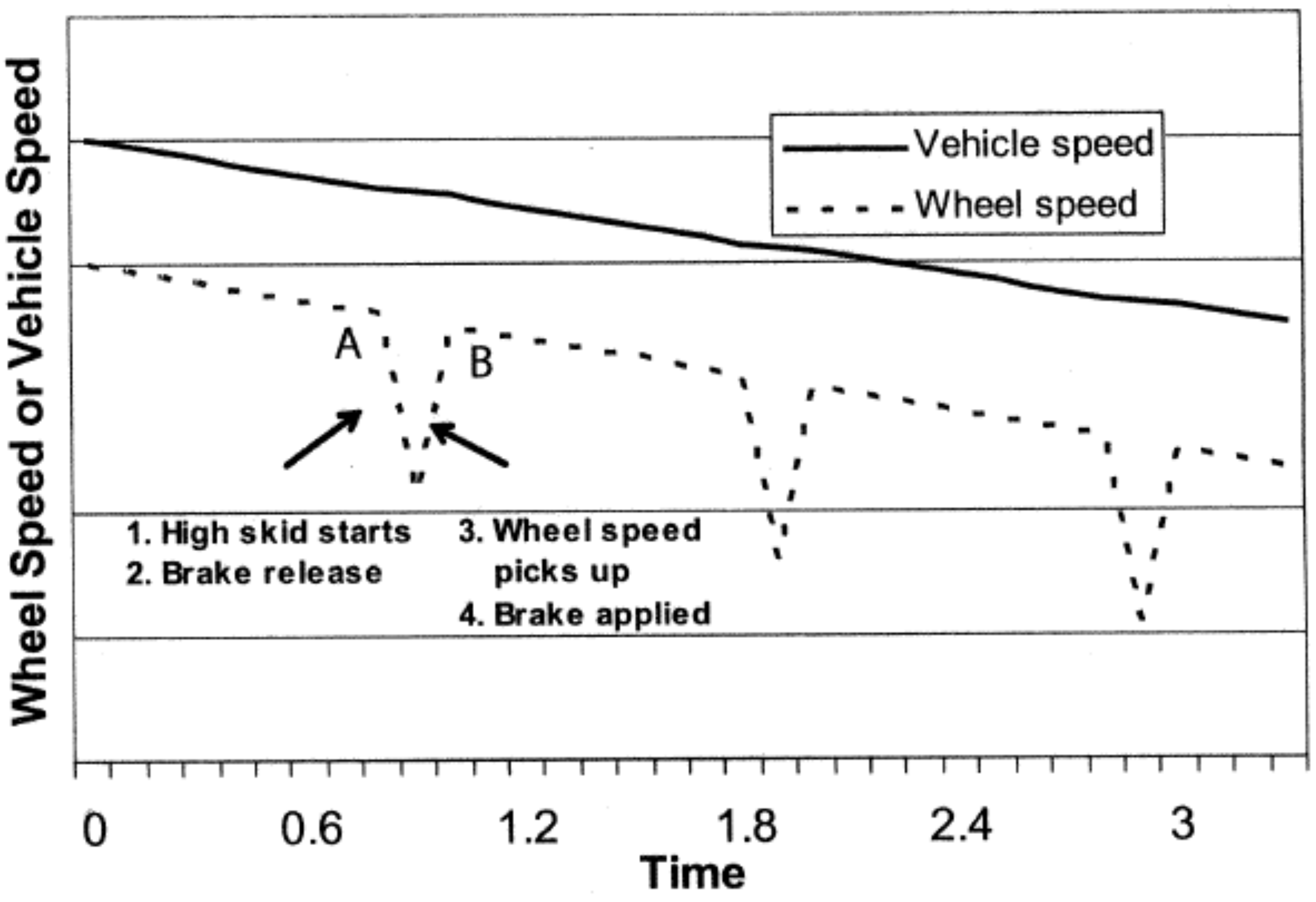
ABS: Introduction



ABS


Anti-Lock Braking System
Speed
Sensor
Pump/Valve
Speed
Sensors
©2000 How Stuff Works

ABS: A simple control strategy



ABS

When the wheels locks you loose

- Brake force
- Stability
- Ability to control the vehicle

The objective of the ABS system is to prevent wheel lock-up

ABS: Detecting wheel lock-up

If the wheels do not slide, then

$$\dot{\omega} r \approx a \leq \mu g$$

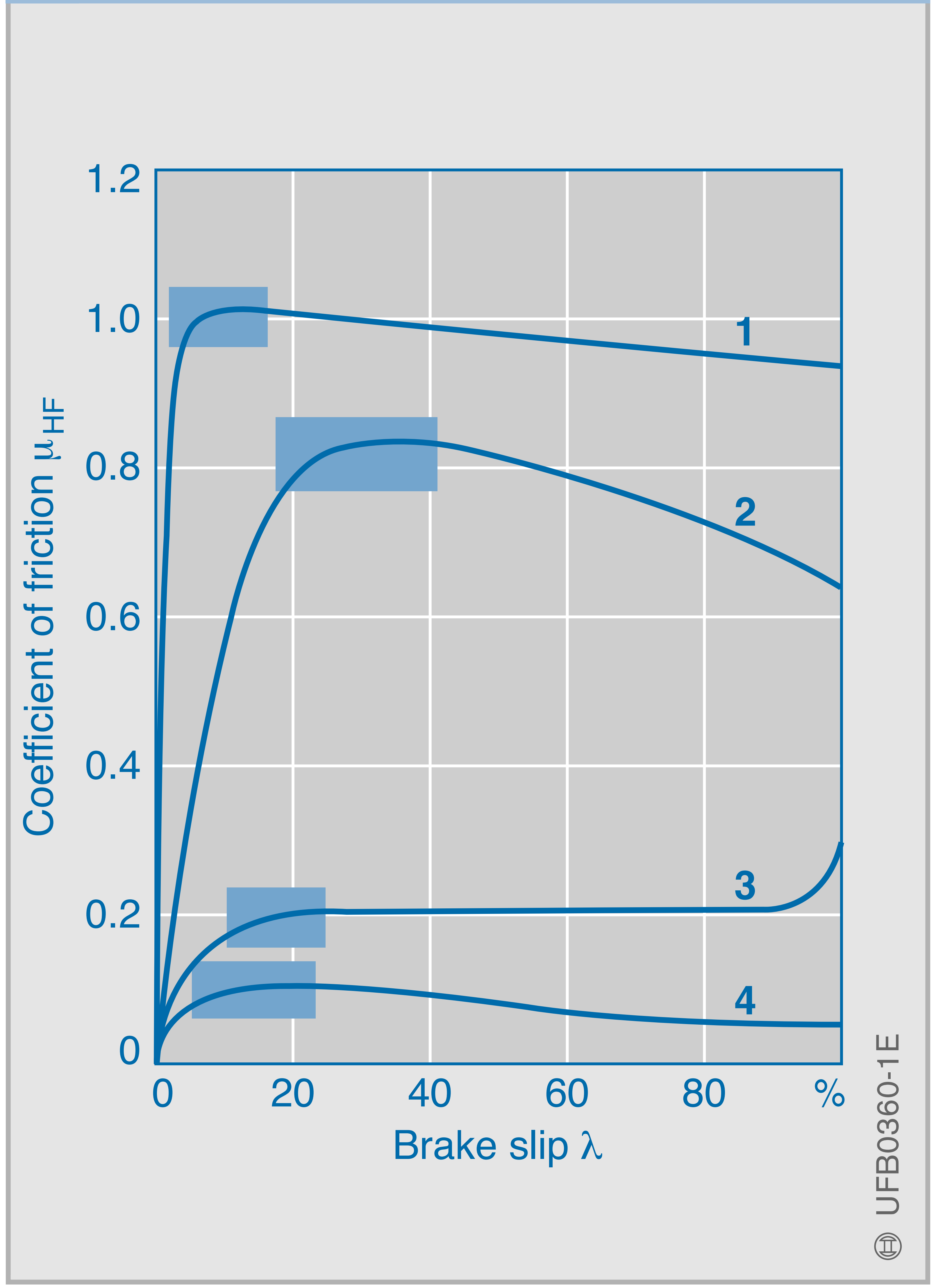
This can be used to detect when the wheels are locking, either by measuring the acceleration a or using an estimate of μ.

Another option is to use the skid

$$i_s = 1 - \frac{r\omega}{V}$$

where V is estimated.

1 Coefficient of friction, μ_{HF}, relative to brake slip, λ



2 Coefficient of friction and lateral-force coefficient, μ_S, versus brake slip, λ, and slip angle

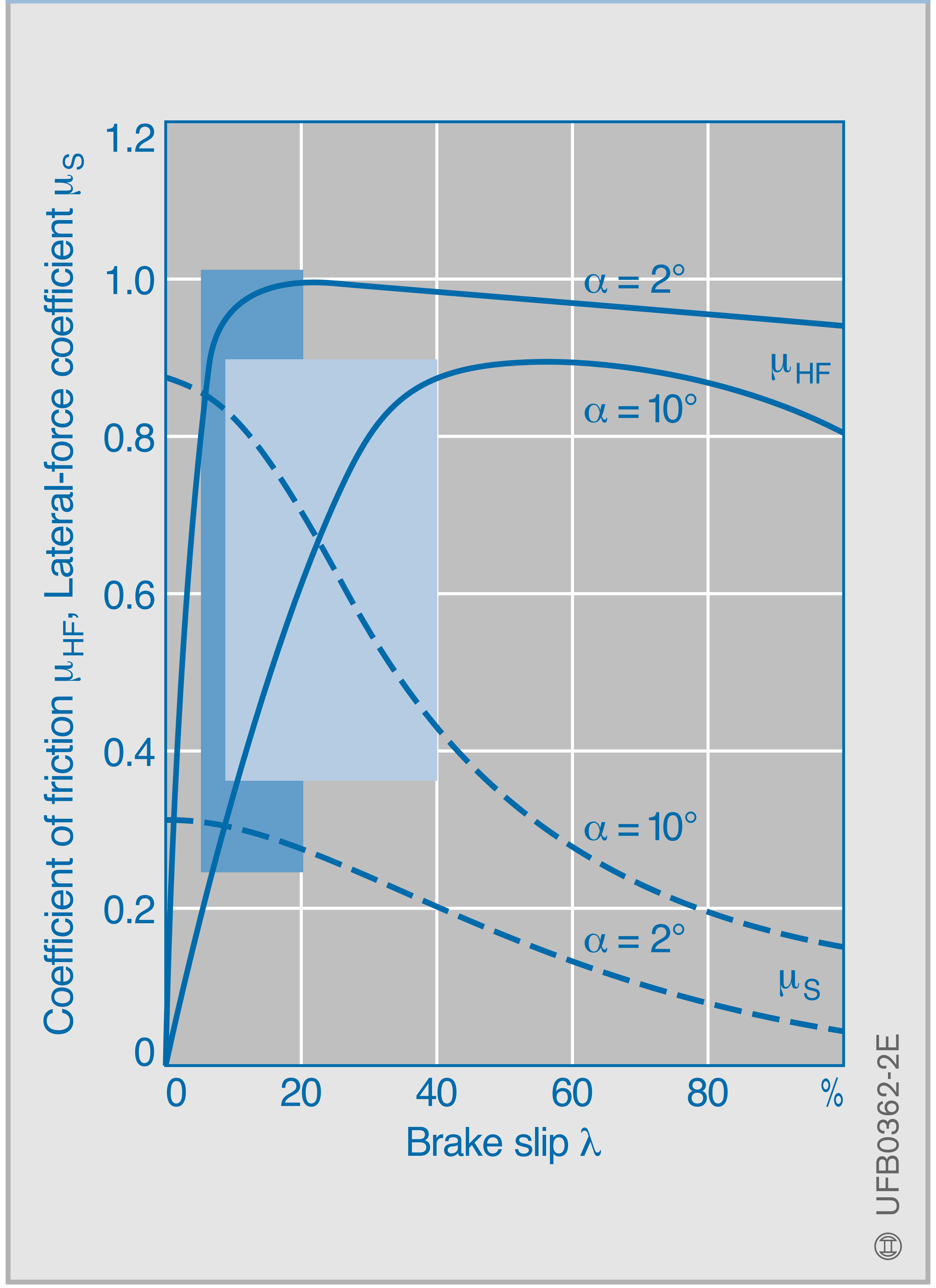


Fig. 1

1 Radial tire on dry concrete
2 Cross-ply tire on wet tarmac
3 Radial tire on loose snow
4 Radial tire on wet black ice

Blue shaded areas: ABS active zones

Fig. 2

μ_{HF} Coefficient of friction
μ_S Lateral-force coefficient
α Slip angle

Blue shaded areas: ABS active zones

3 Braking control cycle for low-adhesion conditions

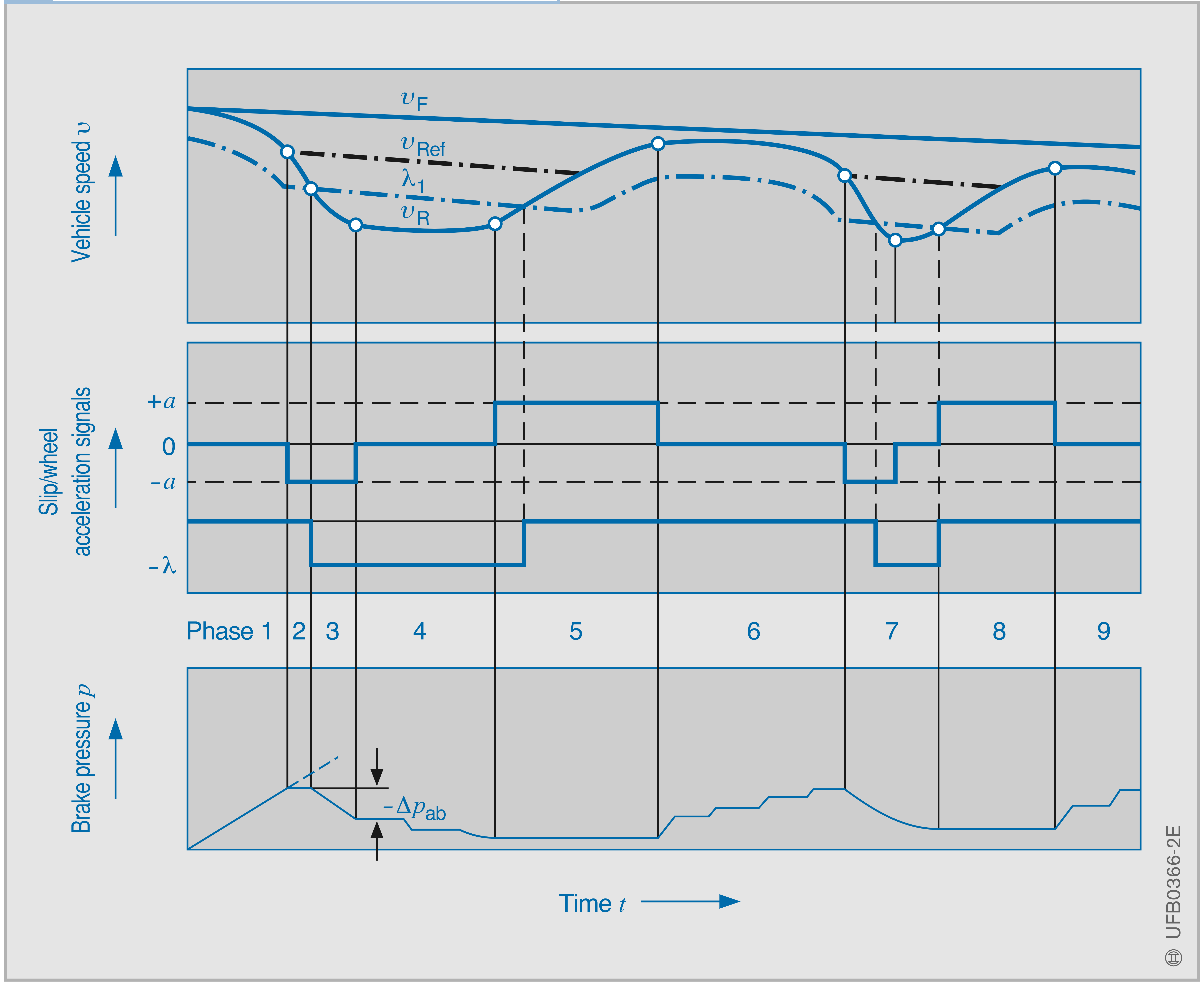


Fig. 3

v_F Vehicle speed

v_{Ref} Reference speed

v_R Wheel speed

λ_1 Slip switching threshold

Switching signals:

$+a$ Threshold for wheel acceleration

$-a$ Threshold for wheel deceleration

$-p_{ab}$ Brake-pressure drop

4 Yaw-moment build-up induced by areas of widely differing adhesion

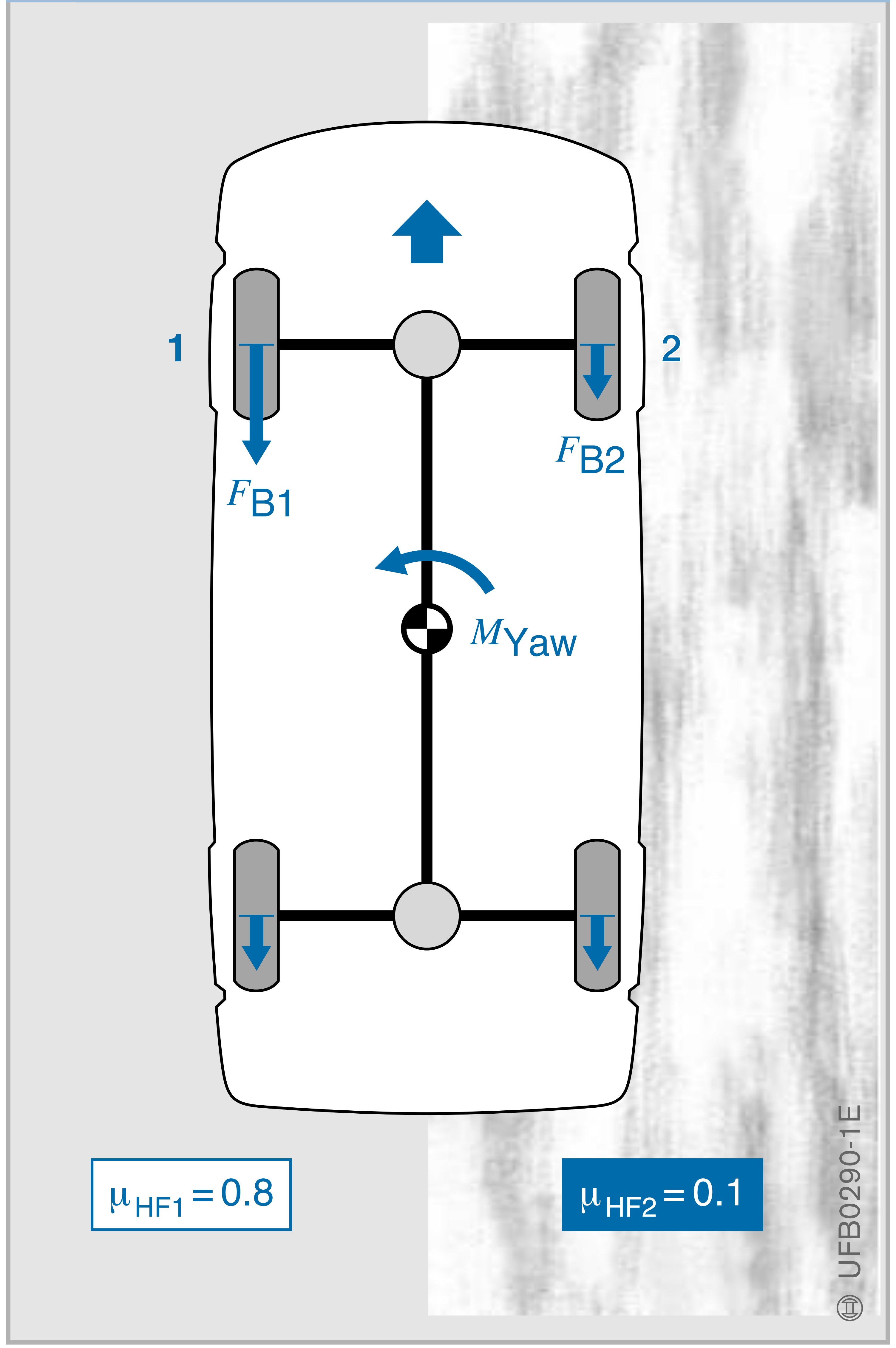


Fig. 4

M_{yaw}	Yaw moment
F_B	Braking force
1	"High" wheel
2	"Low" wheel