Vehicle Dynamics and Control

Lecture 1

LINKÖPING UNIVERSITY

Introduction

Course Literature

Course book is Theory of Ground Vehicles, 4th edition, by J.Y. Wong

You can borrow a copy during the course.

Chapter 1: Mechanics of Pneumatic Tires

Chapter 3: Performance Characteristics of Road Vehicles

Chapter 5: Handling Characteristics of Road Vehicles

Chapter 7: Vehicle Ride Characteristics

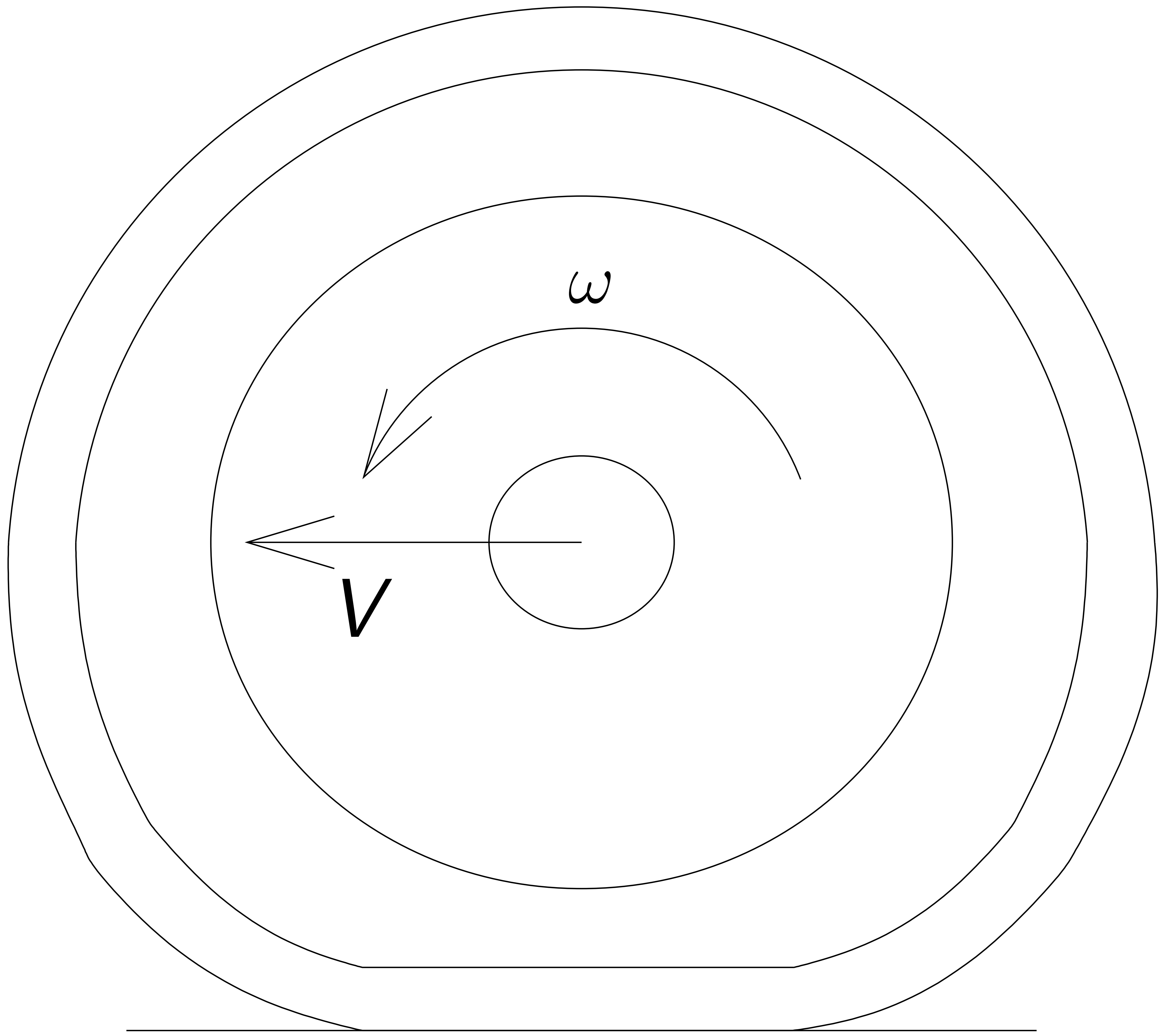
Some additional material is taken from the books

- *Vehicle Dynamics, Stability and Control*, 2nd edition, D. Karnopp
- *Tire and Vehicle Dynamics*, H. Pacejka.

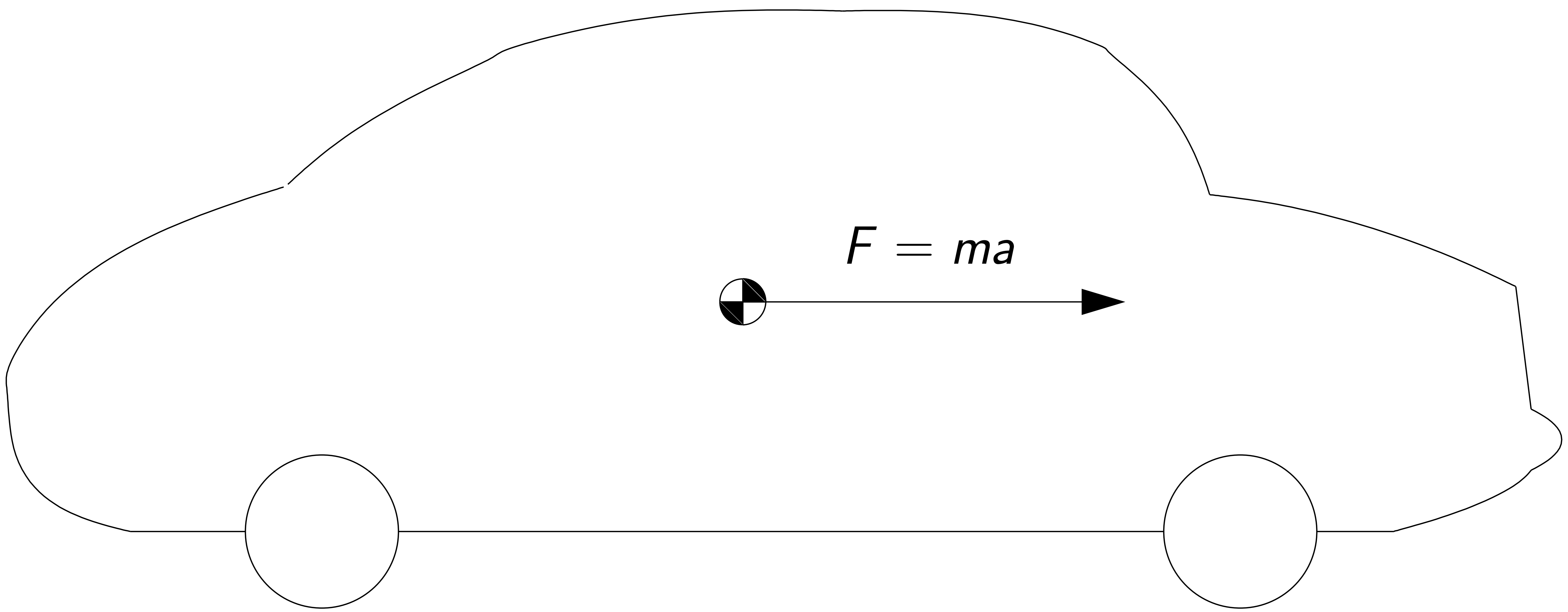
The lectures

- Tyre modelling
- Longitudinal dynamics and control
- Lateral dynamics and control
- Vertical dynamics and control
- Stability and control
- Applications

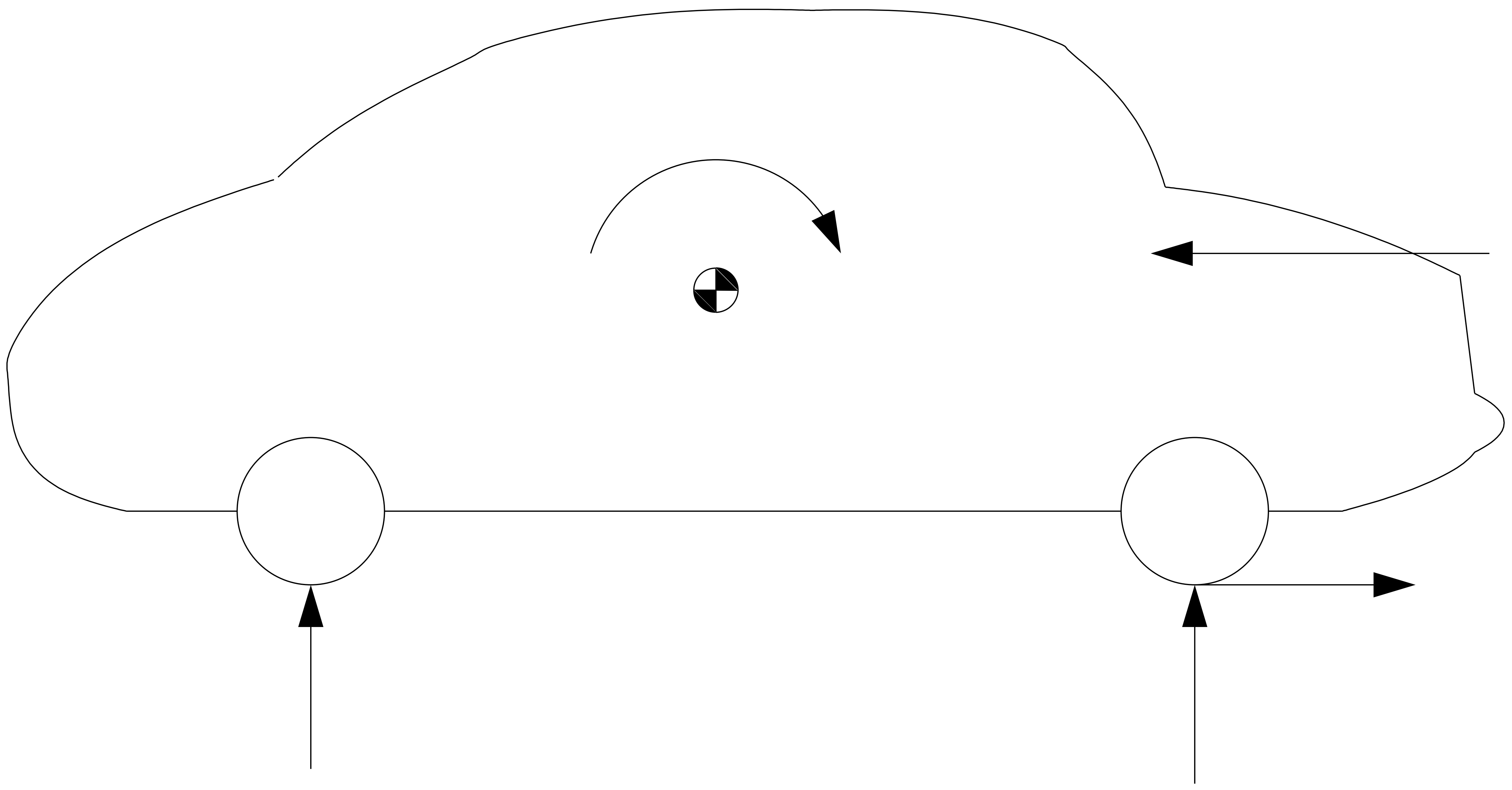
Tyre Modelling:
Mechanics of Pneumatic Tires (chapter 1)



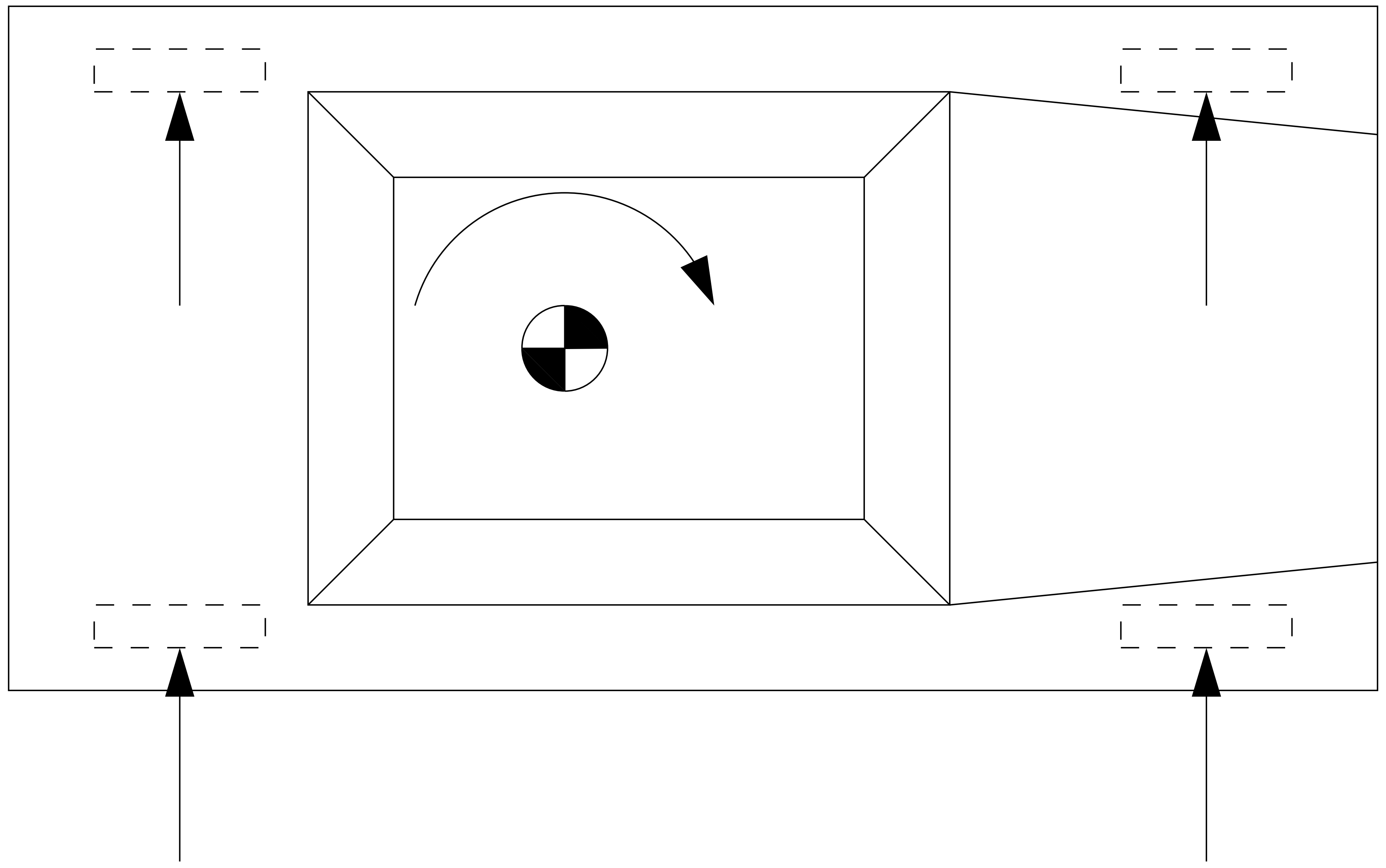
Longitudinal Dynamics: Performance Characteristics of Road Vehicles (ch.3)



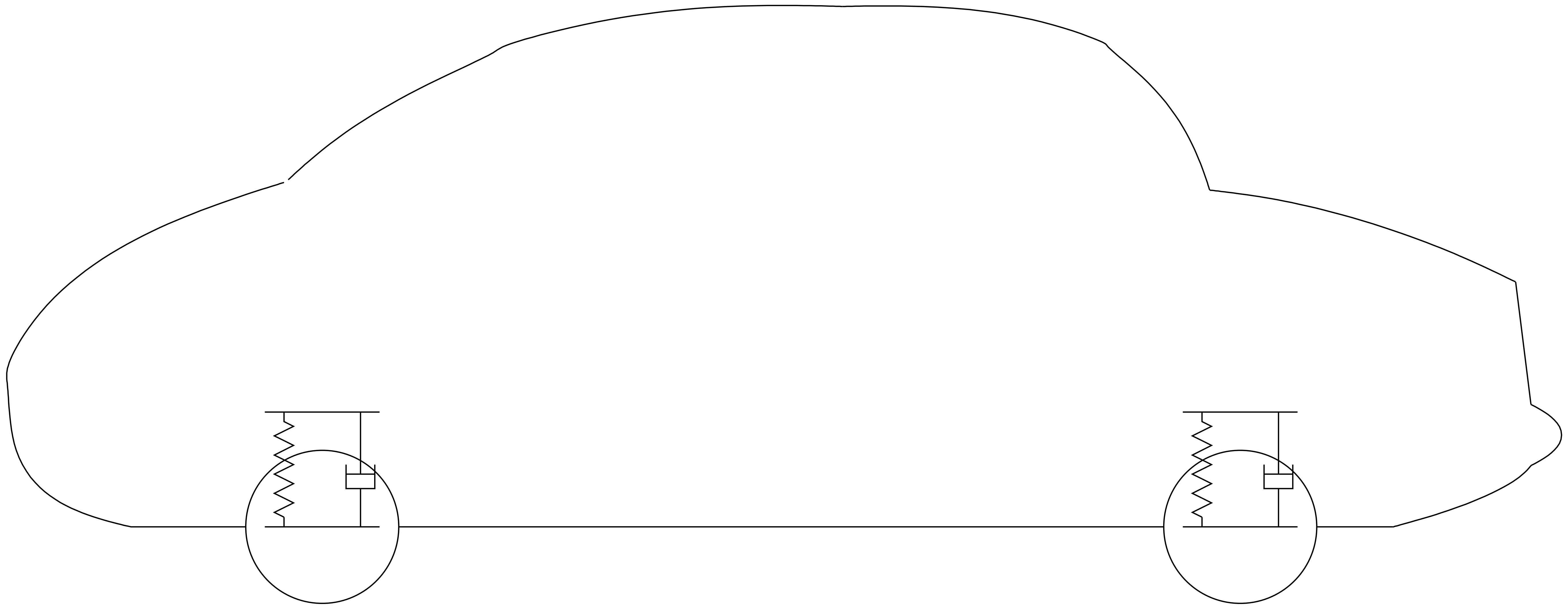
Longitudinal Dynamics: Performance Characteristics of Road Vehicles (ch.3)



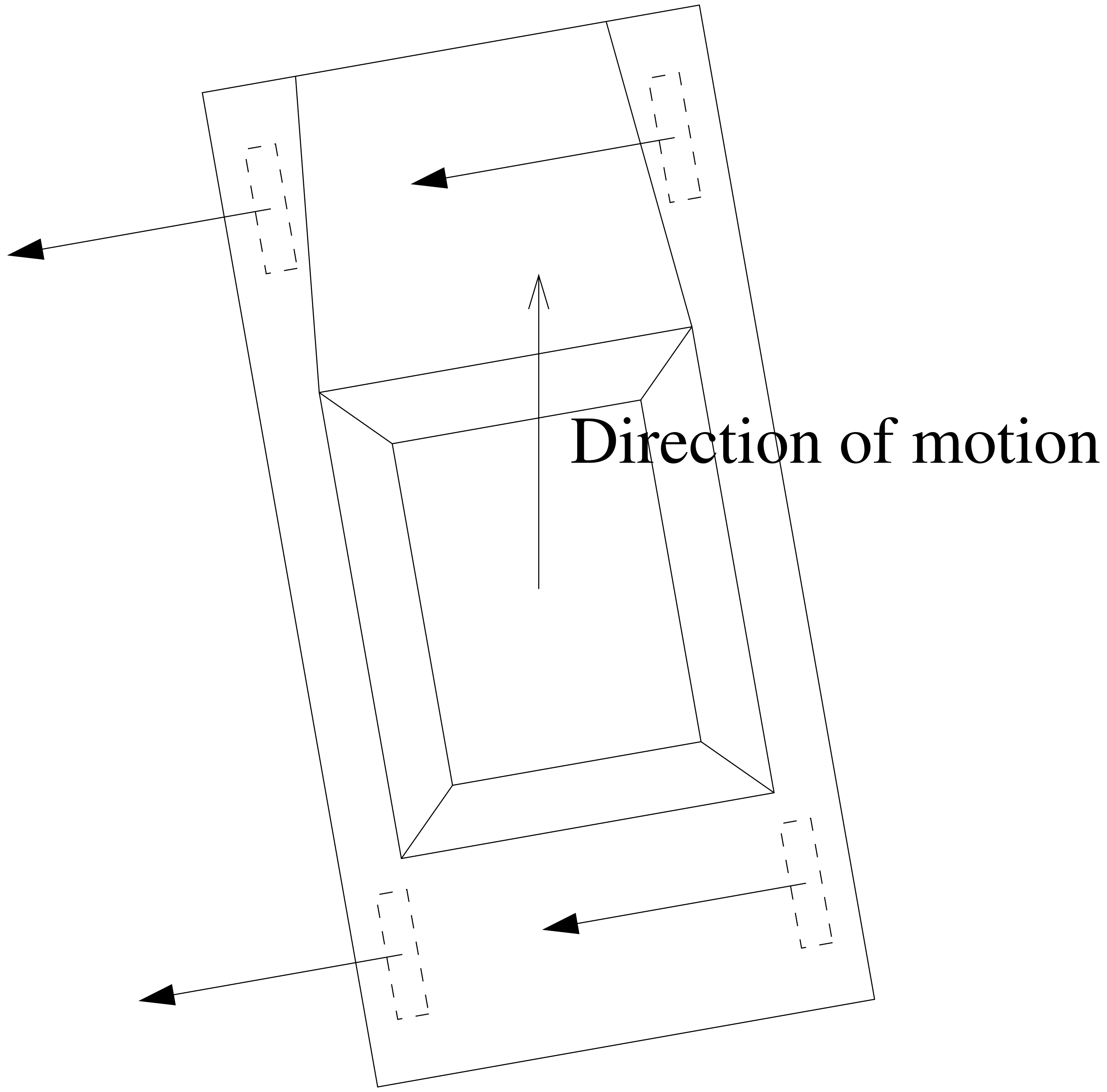
Lateral dynamics:
Handling Characteristics of Road Vehicles (ch.5)



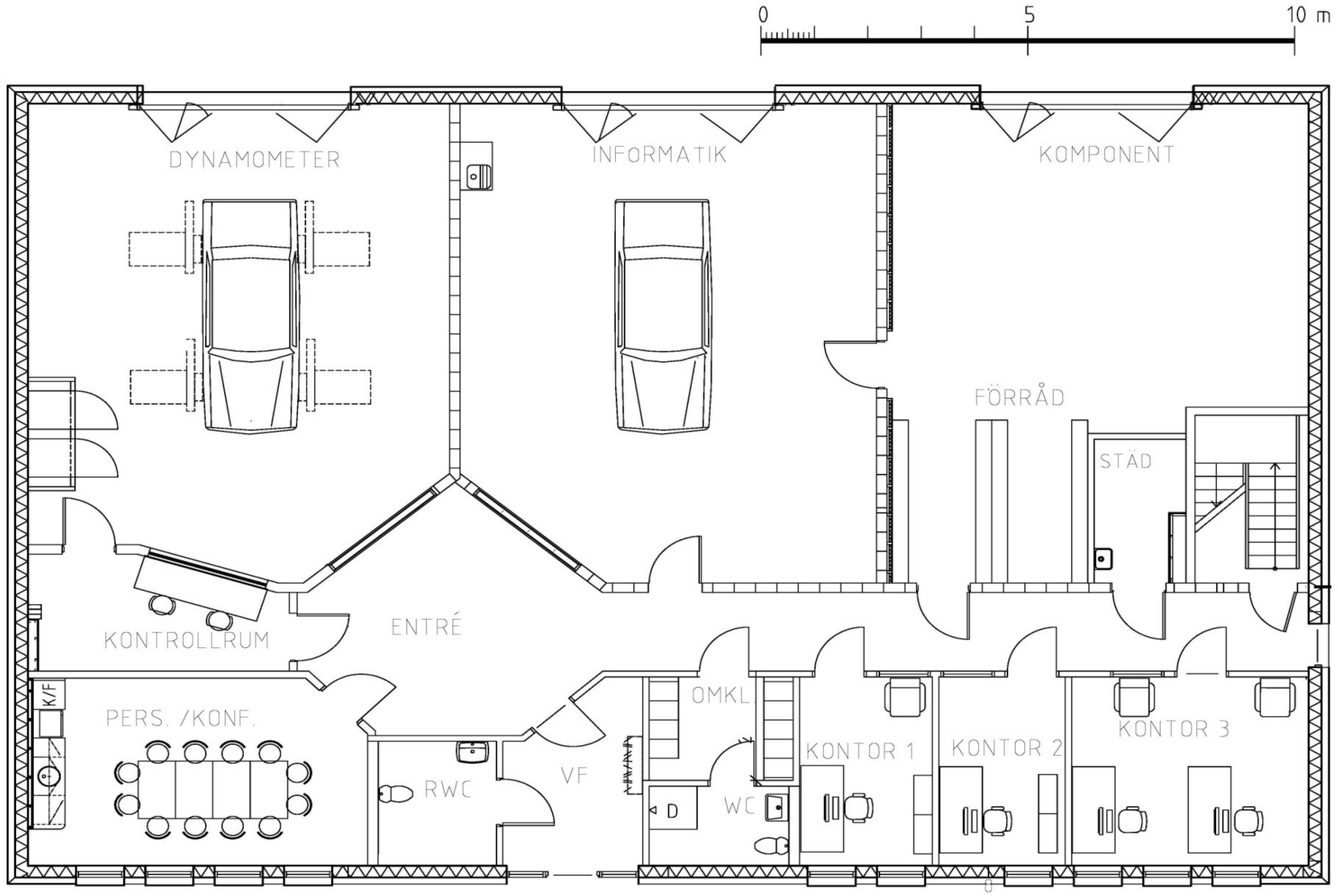
Vertical Dynamics:
Vehicle Ride Characteristics (ch.7)



Stability



L-building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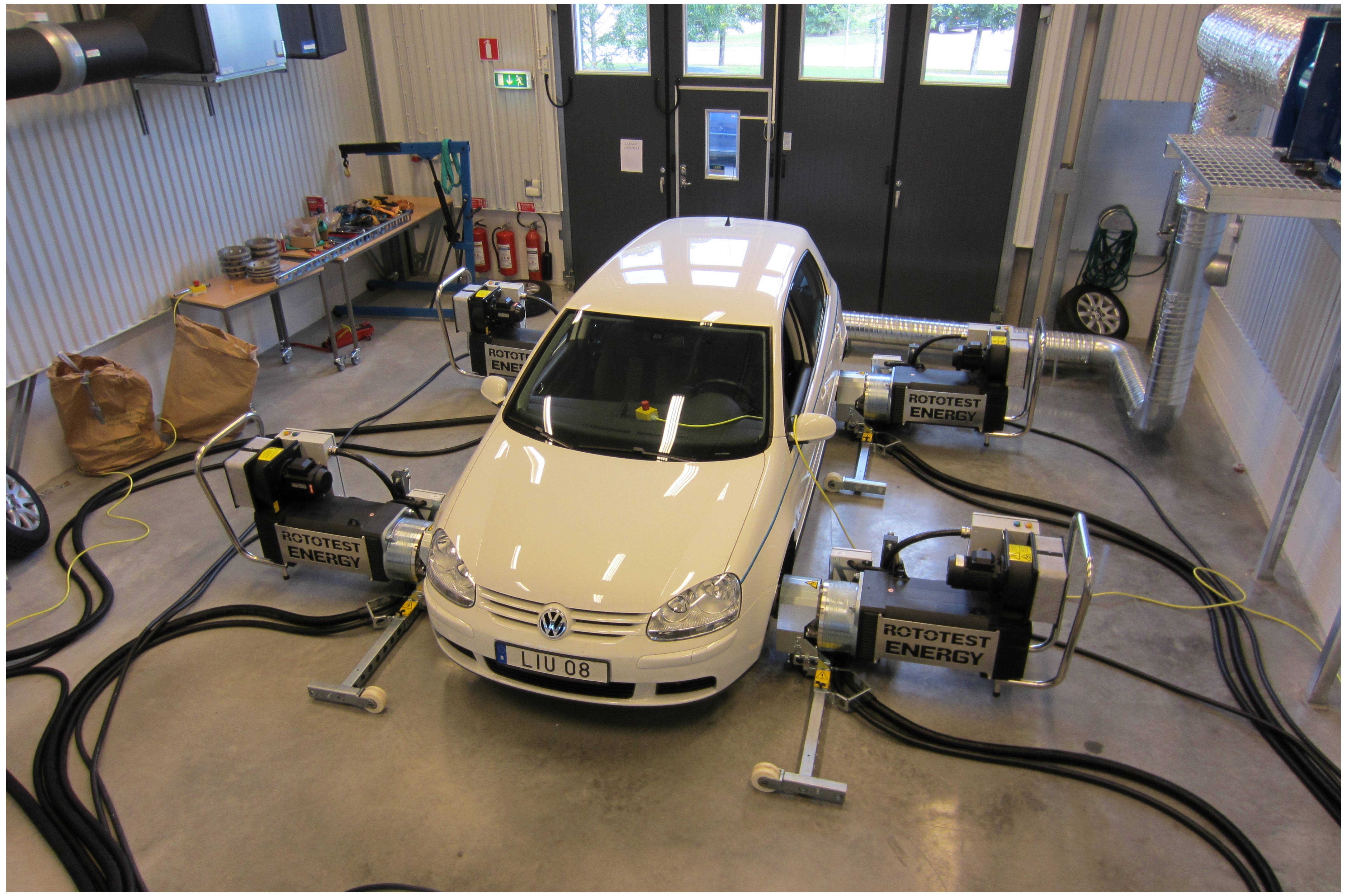
L-building



L-building

Sensor Set-up

1. Slip angle sensor
2. Pitch/roll angle sensors
3. IMU
4. GPS
5. Vehicle CAN

Today's lecture

- Stability: Tapered wheels on trains
- Tyre modelling: Rolling resistance
- Tyre modelling: The brush model

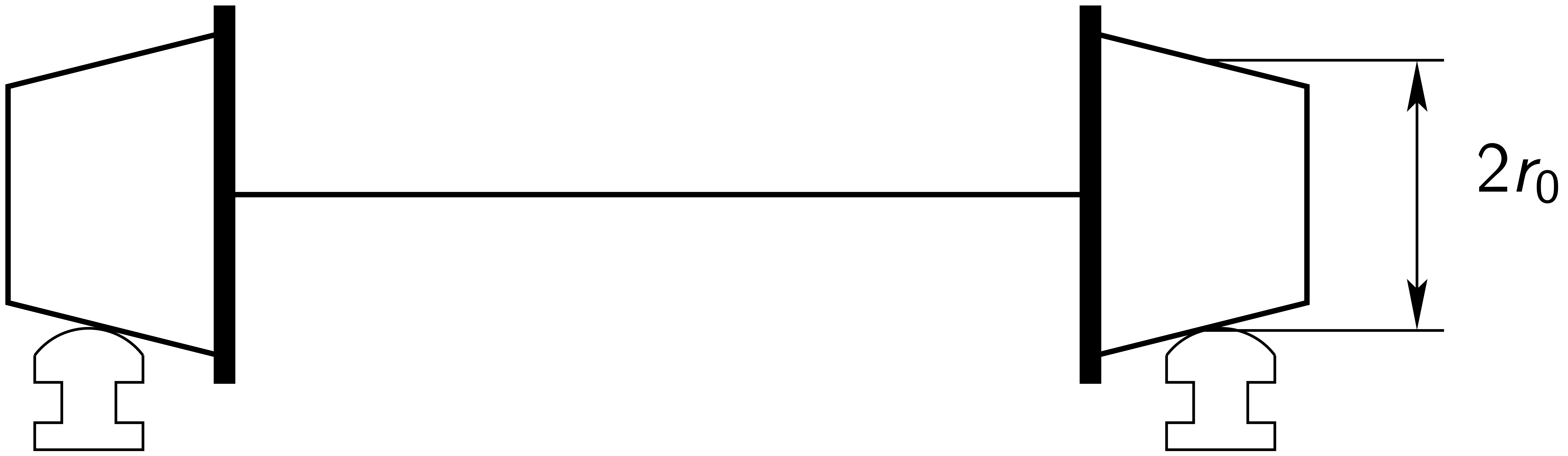
Stability: Tapered Wheels on Trains

Tapered Wheels on Trains

Why are the wheels on a train tapered (sw. konformade)?

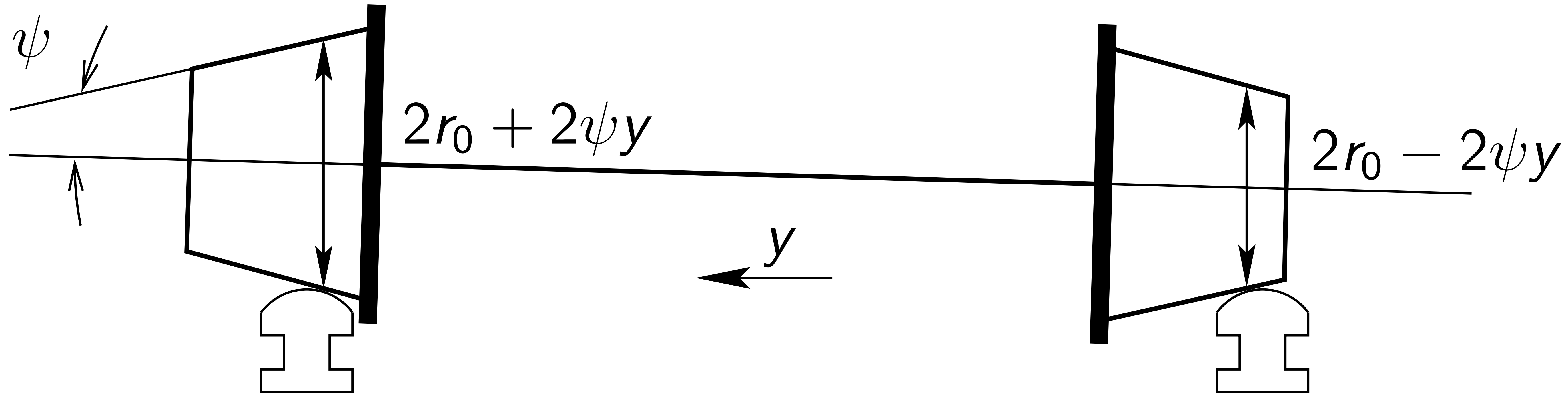
Tapered wheels: Basic motion

Consider a wheelset with tapered wheels on a rail. In the **steady motion/basic motion**, the wheels are moving on a straight line in the **longitudinal** direction:



Tapered wheels: A train taking a turn

One reason for using tapered wheels is illustrated in the following figure showing a wheelset of a train taking a right turn:



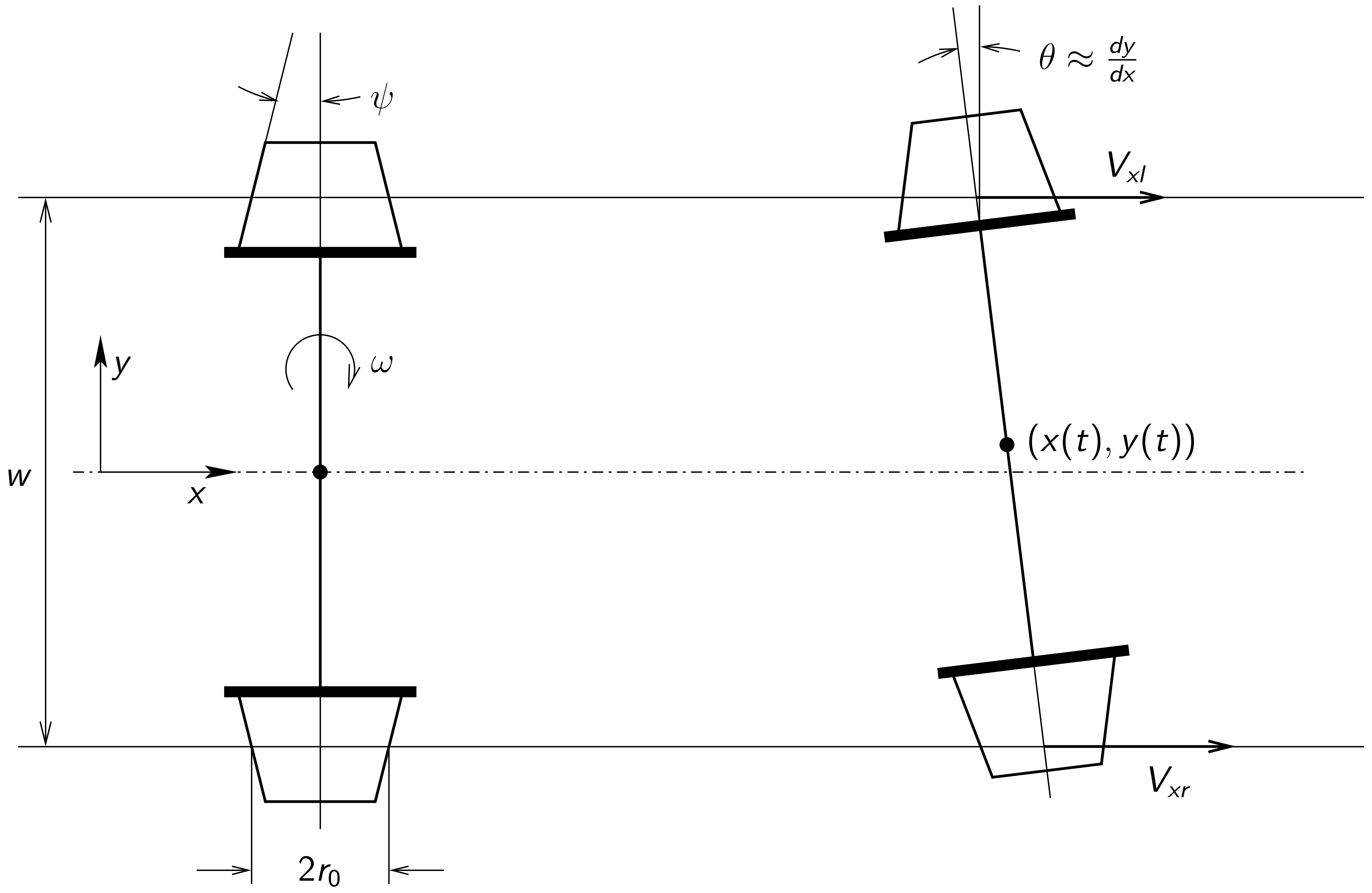
The longitudinal speed is larger for the outside wheel V_{xl} than for the inside wheel V_{xr}, but the rotational speed ω is the same. The basic motion in this case includes a constant drift y in the **lateral** direction, which compensates for this difference:

$$V_{xl} = (r_0 + \psi y)\omega, \quad V_{xr} = (r_0 - \psi y)\omega$$

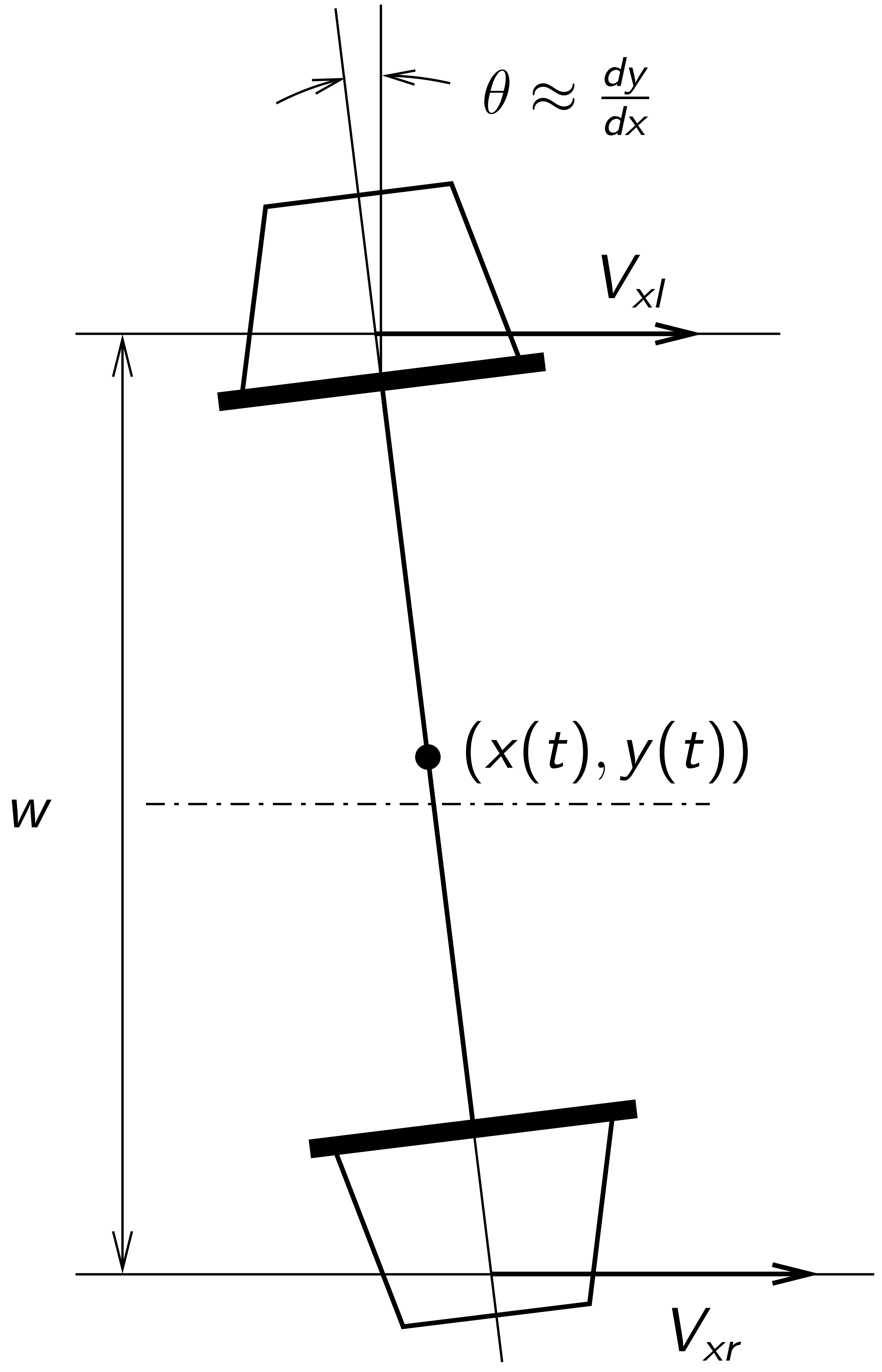
Tapered wheels: Perturbed motion

What will happen if the basic motion is perturbed?

Basic motion is shown to the left and perturbed motion to the right:



Tapered wheels



As before, lateral drift causes a difference in the longitudinal velocity of the wheels:

$$V_{xl} = (r_0 + \psi y)\omega$$
$$V_{xr} = (r_0 - \psi y)\omega$$

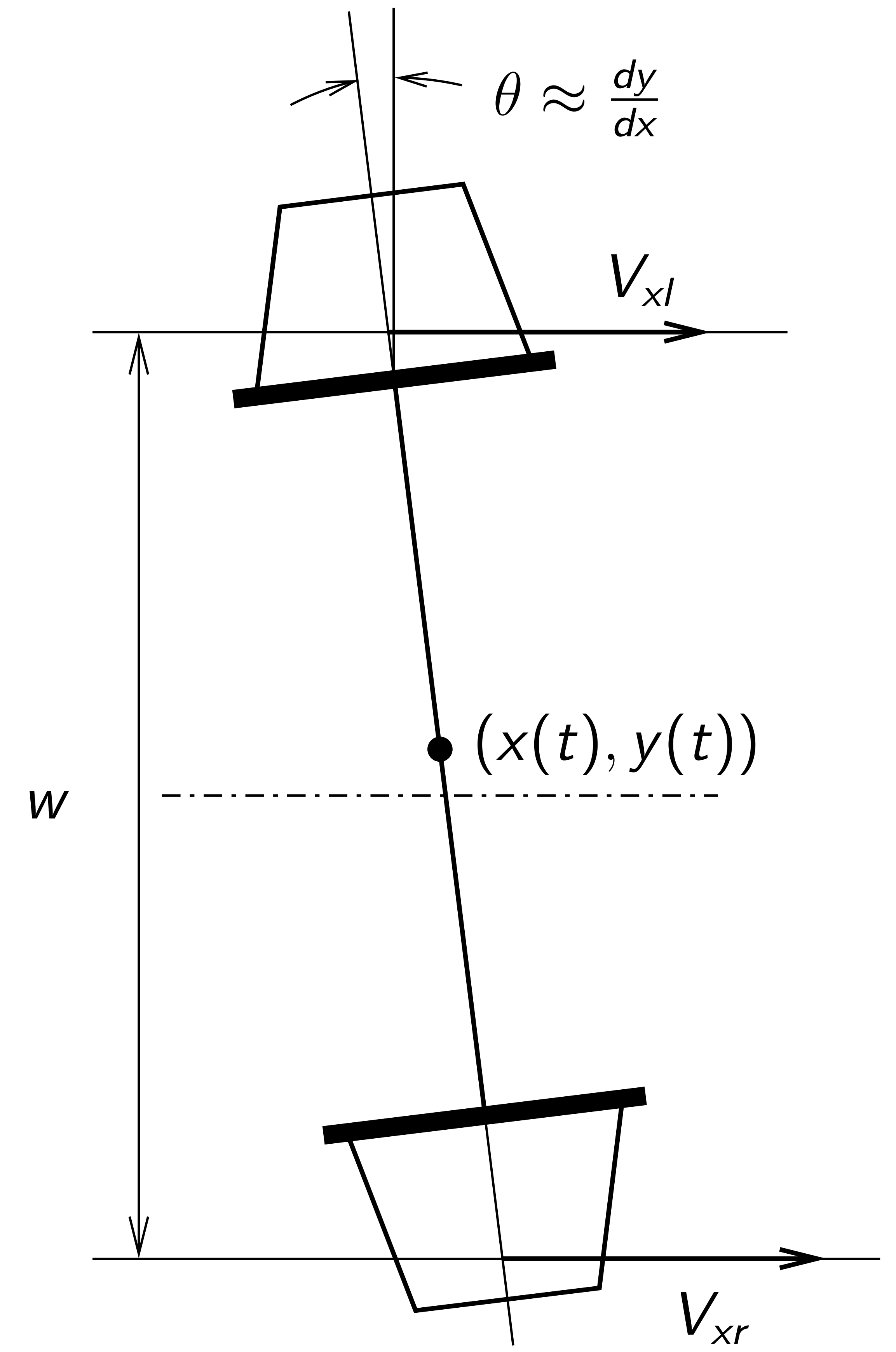
The longitudinal velocity of the center of gravity is now given by:

$$\dot{x} = \frac{V_{xl} + V_{xr}}{2} = r_0\omega$$

The approximation $\frac{dy}{dx} \approx \theta$ gives the lateral velocity:

$$\dot{y} = \frac{dy}{dx}\frac{dx}{dt} = \theta r_0 \omega$$

Tapered wheels



Using $V_{xl} = (r_0 + \psi y)\omega$ and $V_{xr} = (r_0 - \psi y)\omega$ the angular velocity can be written as

$$\dot{\theta} = \frac{V_{xr} - V_{xl}}{w} = -\frac{2\psi y\omega}{w}$$

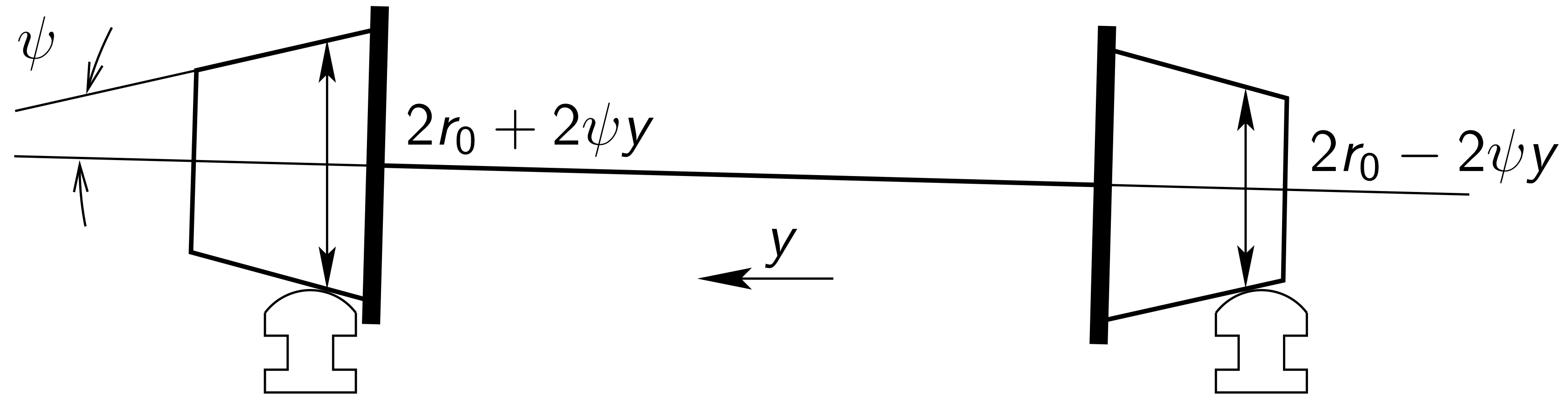
Differentiating $\dot{y} = \theta r_0 \omega$ and using the expression for the angular velocity above, the following differential equation for y is obtained:

$$\ddot{y} + \frac{2r_0\psi\omega^2}{w} y = 0$$

Tapered wheels: Harmonic oscillation

For a wheelset with positive taper angle (as in the figure) the solution of

$$\ddot{y}(t) + \frac{2r_0\psi\omega^2}{w} y(t) = 0$$



is a harmonic oscillation

$$y(t) = \cos(\omega_n t + \phi)$$

with natural frequency

$$\omega_n = \sqrt{\frac{2r_0\psi}{w}}\omega$$

If there is friction in the system, then the wheelset will return to the basic motion asymptotically, and the system is **stable**.

Tapered wheels: Unstable system

For a wheelset with negative taper angle the solutions of the differential equation

$$\ddot{y}(t) + \frac{2r_0\psi\omega^2}{w}y(t) = 0$$

are

$$y(t) = C\exp\left(\pm\sqrt{\frac{2r_0\psi}{w}}\omega\right)$$

which means that the a small perturbation would cause an exponential growth of the lateral displacement, and the system is clearly **unstable**.

Tapered wheels: Spatial coordinates

The dynamic equation

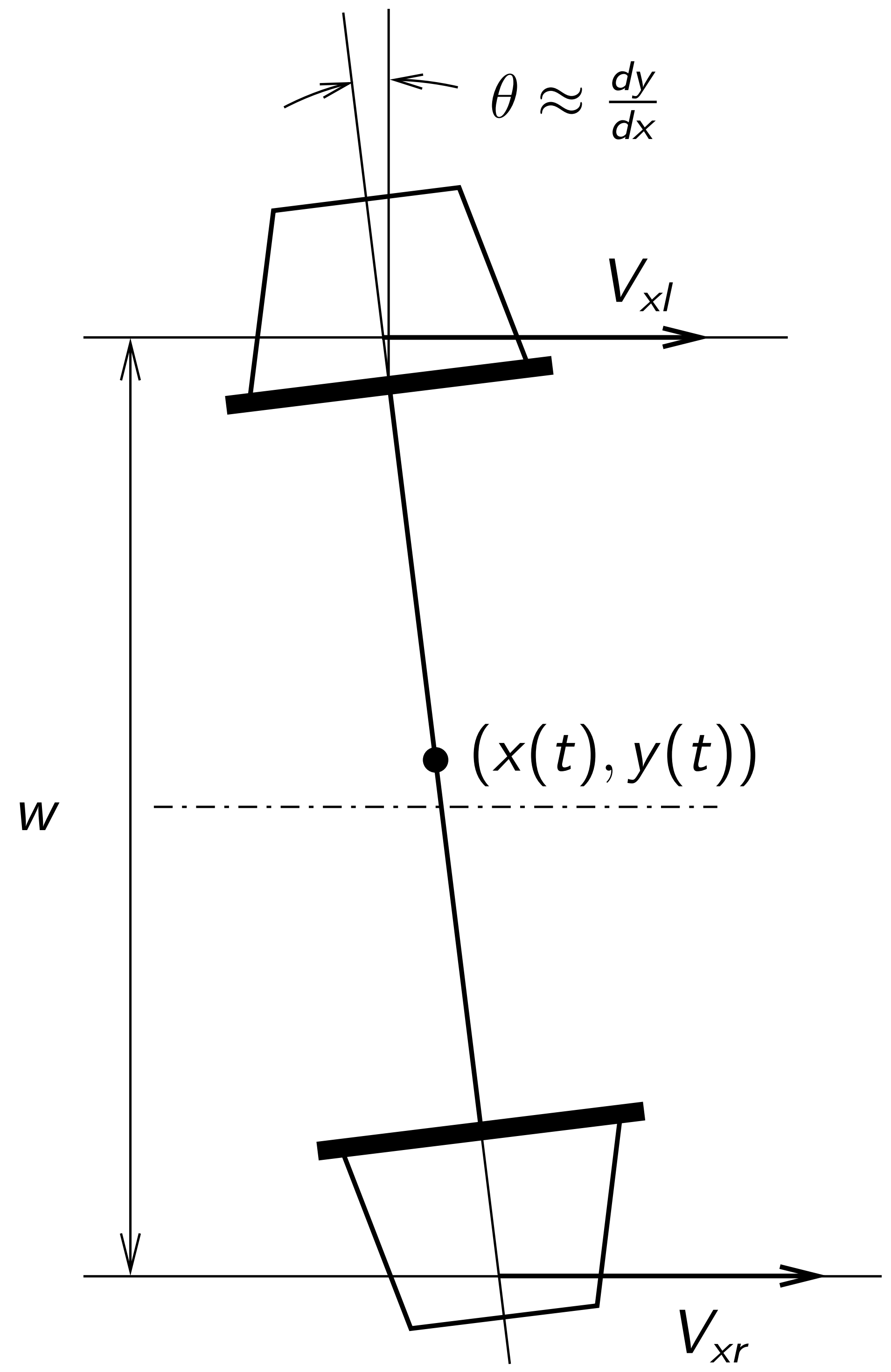
$$\ddot{y}(t) + \frac{2r_0\psi\omega^2}{w}y(t) = 0$$

can be rewritten by using the relations

$$\ddot{y} = \frac{d^2y}{dx^2}\dot{x}^2, \quad \omega^2 = \frac{\dot{x}^2}{r_0^2}$$

and the result is the following:

$$y''(x) + \frac{2\psi}{wr_0}y(x) = 0$$



The model that doesn't depend on speed, with the following curve as solution:

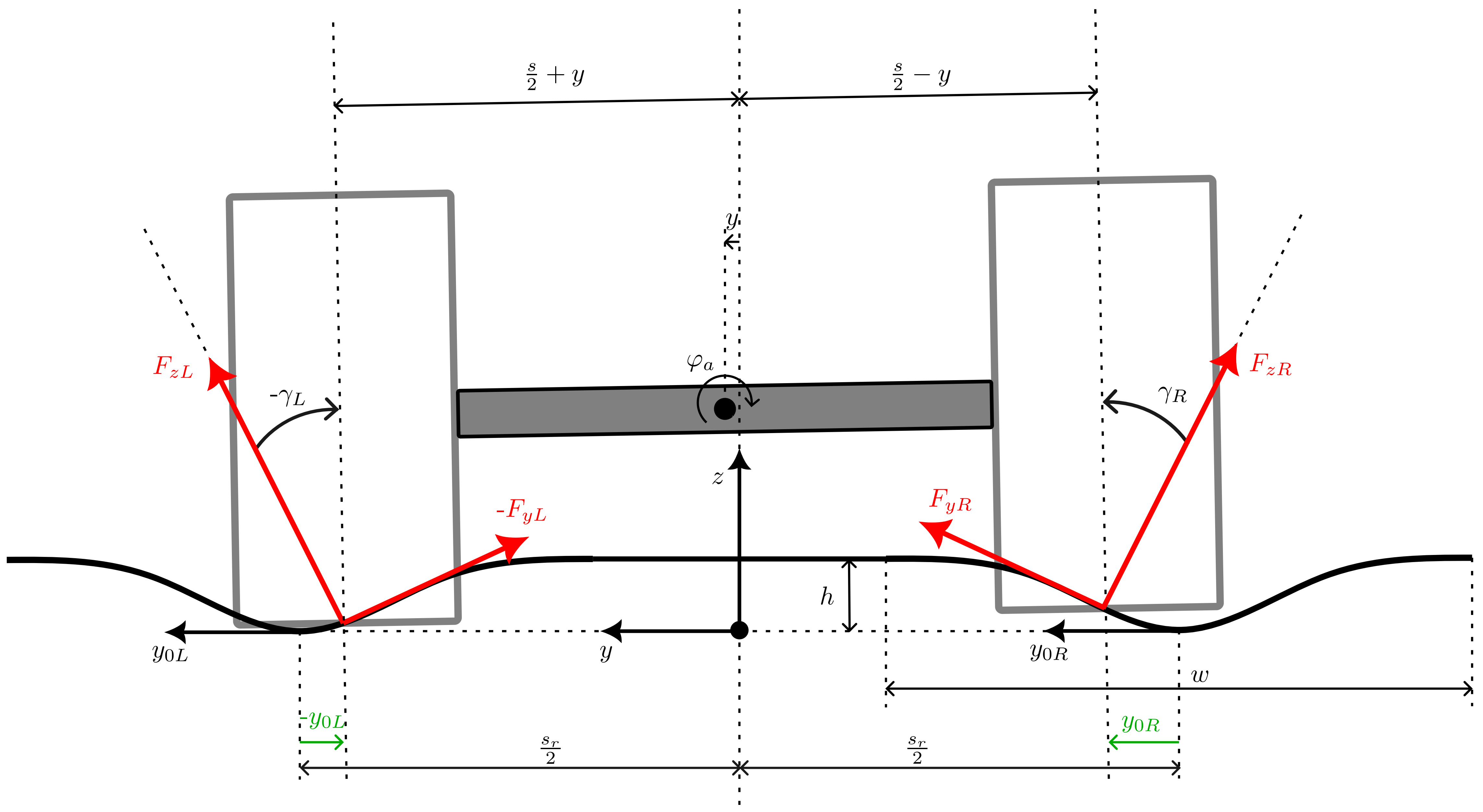
$y(x) = \cos(\omega_s x + z)$ where $\omega_s = \sqrt{\frac{\psi}{wr_0}}$

A Similar Problem

Figure from the paper

Straight line stability of tractor semi-trailer combinations on rutted roads

Sandeep Santhosh Nair, and Igo J.M. Besselink



The unstable case

Tyre Modeling: Rolling Resistance

Tire

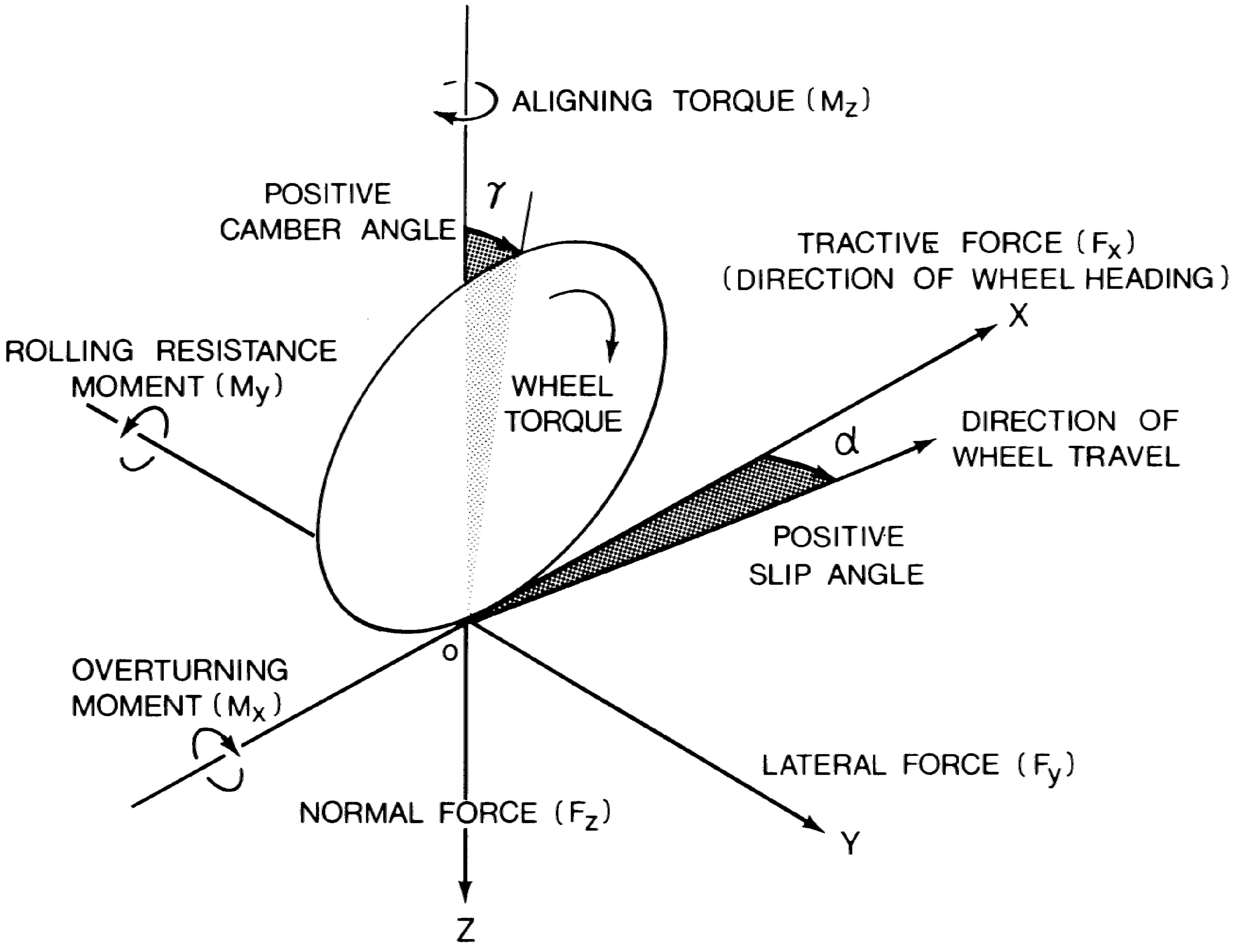


Figure 1.2: Coordinates, forces, and moments.

Rolling resistance

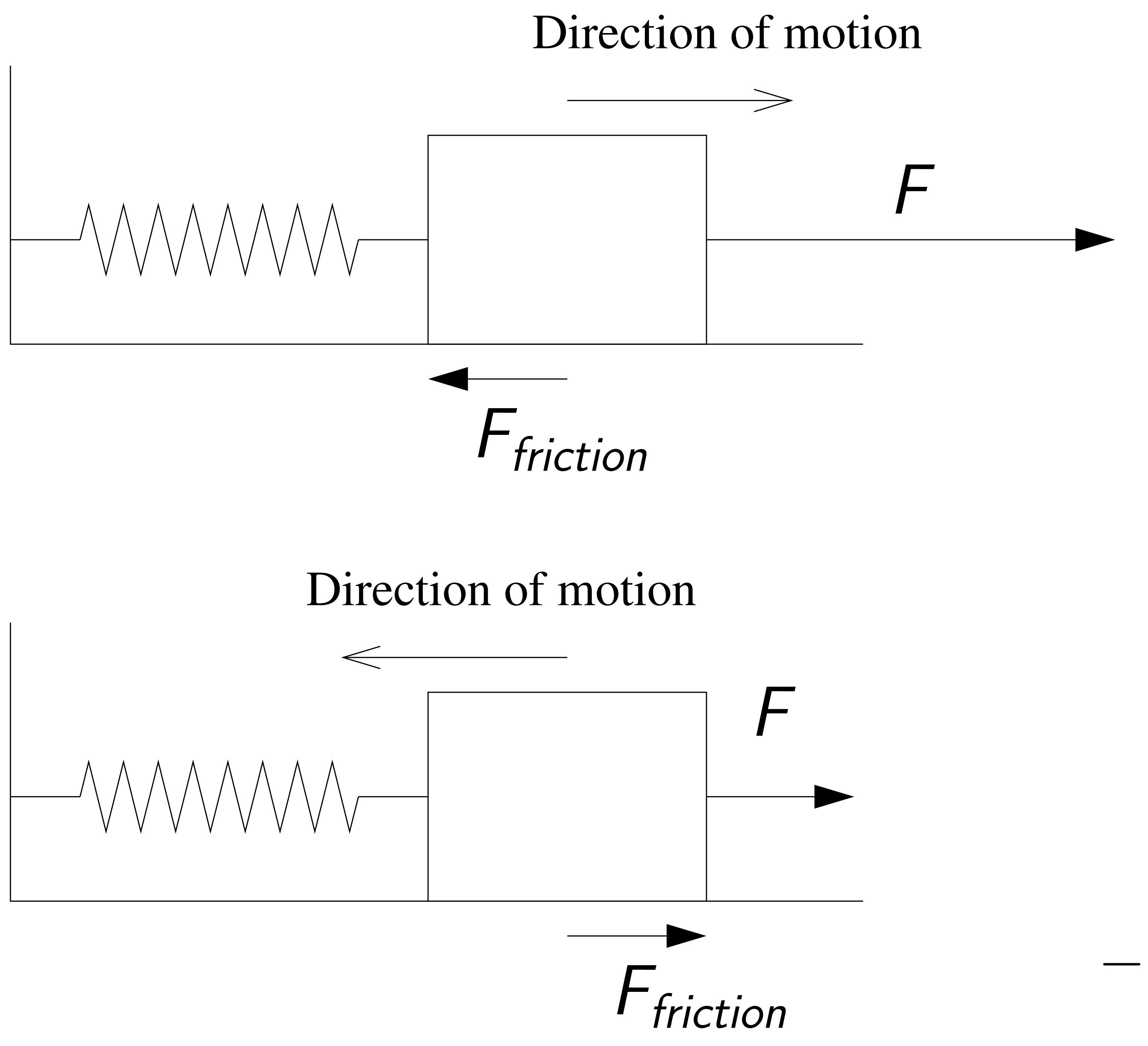
The rolling resistance of tires is primarily caused by the hysteresis in tire materials due to the deflection of the carcass while rolling.

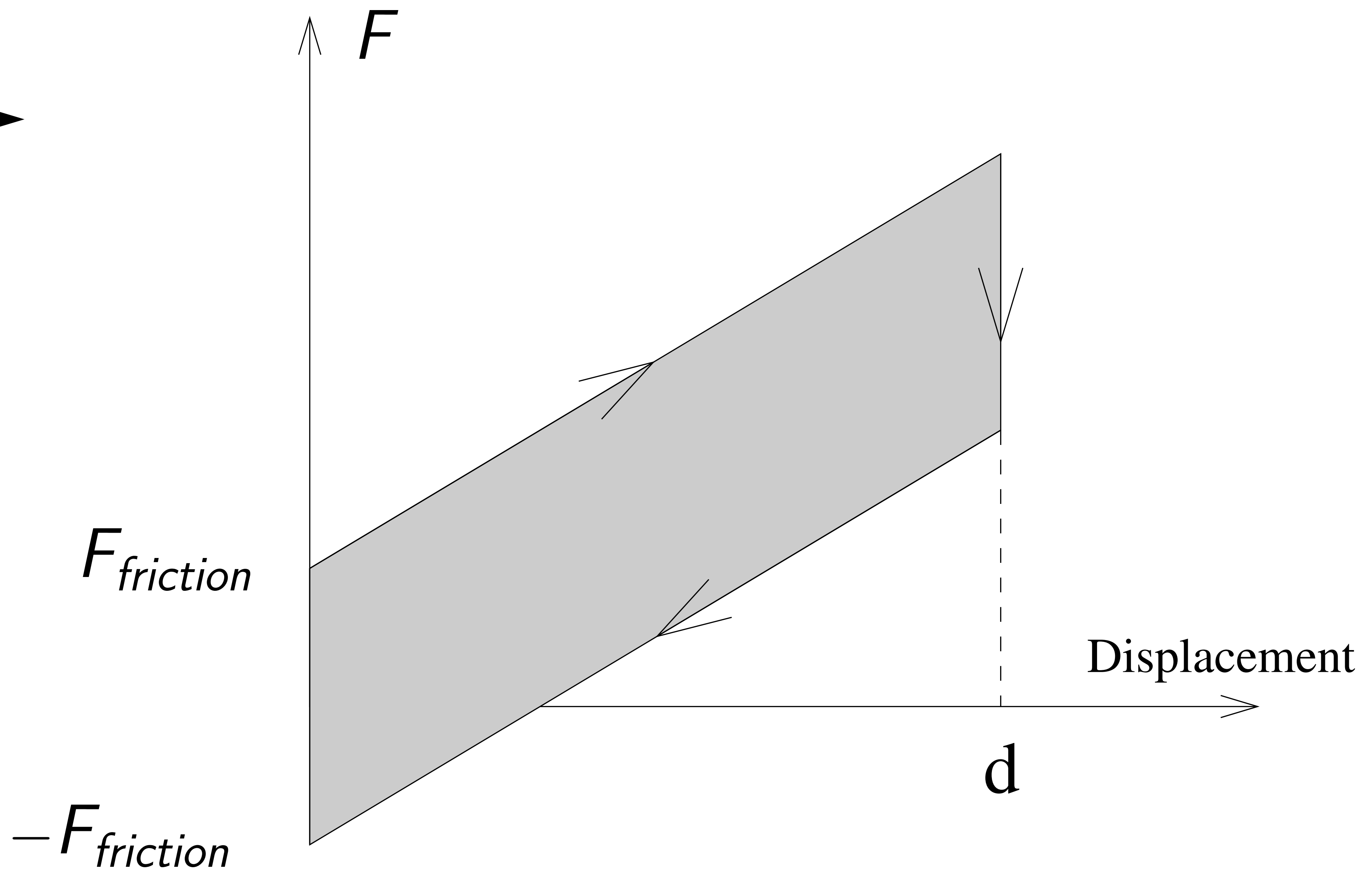
Other less important contributors to the rolling resistance are:

- Friction between the tire and the road caused by sliding
- Air circulating inside the tire

Hysteresis

Exampel of a hysteresis loop caused by friction:

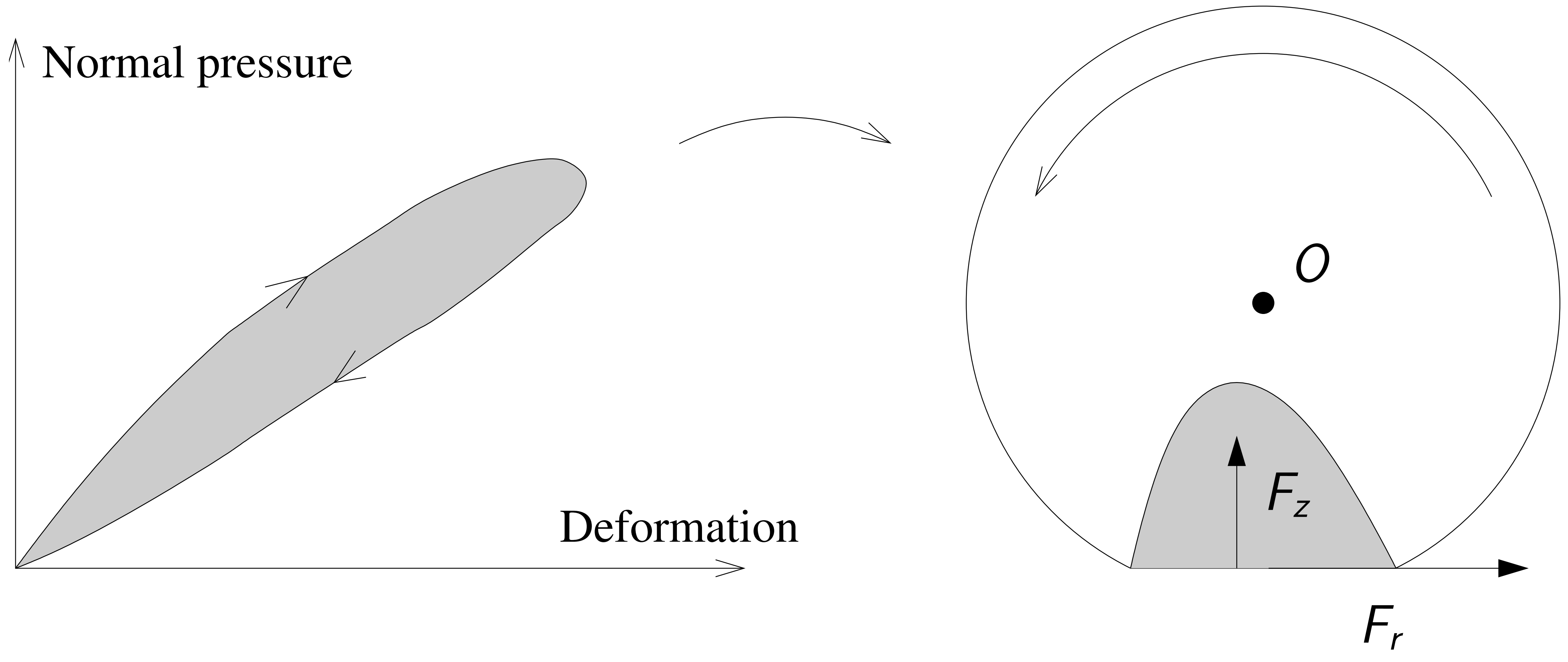




The energy loss due to hysteresis is equal to the shaded in the figure: $2 \cdot d \cdot F_{friction}$

Rolling resistance: Hysteresis

The center of normal pressure is shifted in the direction of motion due to the hysteresis.



The applied wheel torque on free-rolling tire is zero. Therefore, a horizontal force R_r at the contact patch must exists to maintain equilibrium. This force is called rolling resistance.

Rolling resistance

The coefficient of rolling resistance f_r is defined as the ratio of the rolling resistance R_r to the normal load W, i.e., $f_r = R_r/W$.

Empirical formulas for calculating the rolling resistance coefficient as a function of speed V, based on experimental data:

Radial-ply passenger car tire: $f_r = 0.0136 + 0.40 \times 10^{-7}V^2$

Radial-ply truck tire: $f_r = 0.006 + 0.23 \times 10^{-6}V^2$

Rolling resistance

Other factors that affect the rolling resistance: Surface texture, inflation pressure, and internal temperature..

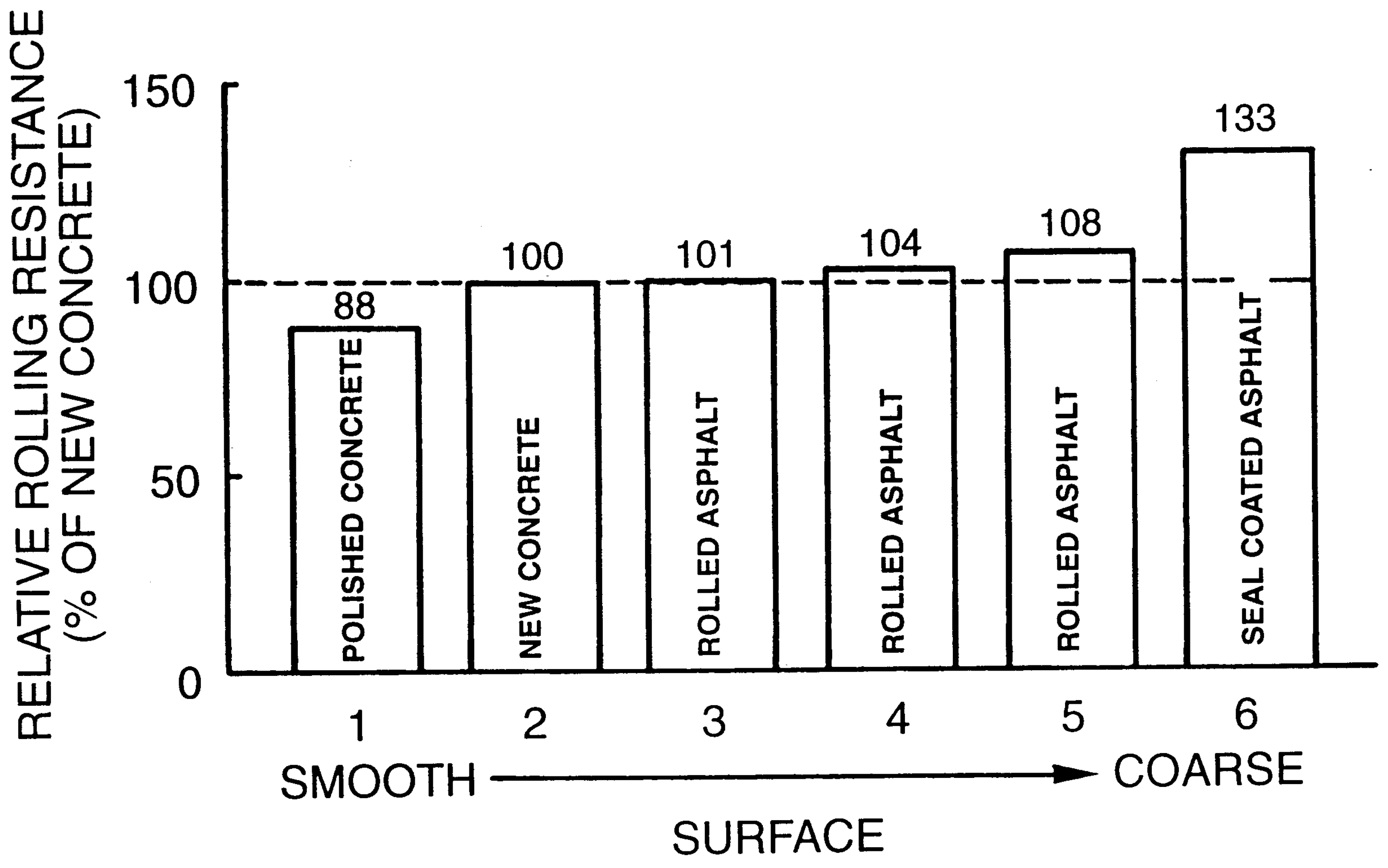


Fig. 1.5 Variation of tire rolling resistance with pavement surface texture. (Reproduced with permission of the Society of Automotive Engineers from reference 1.10.)

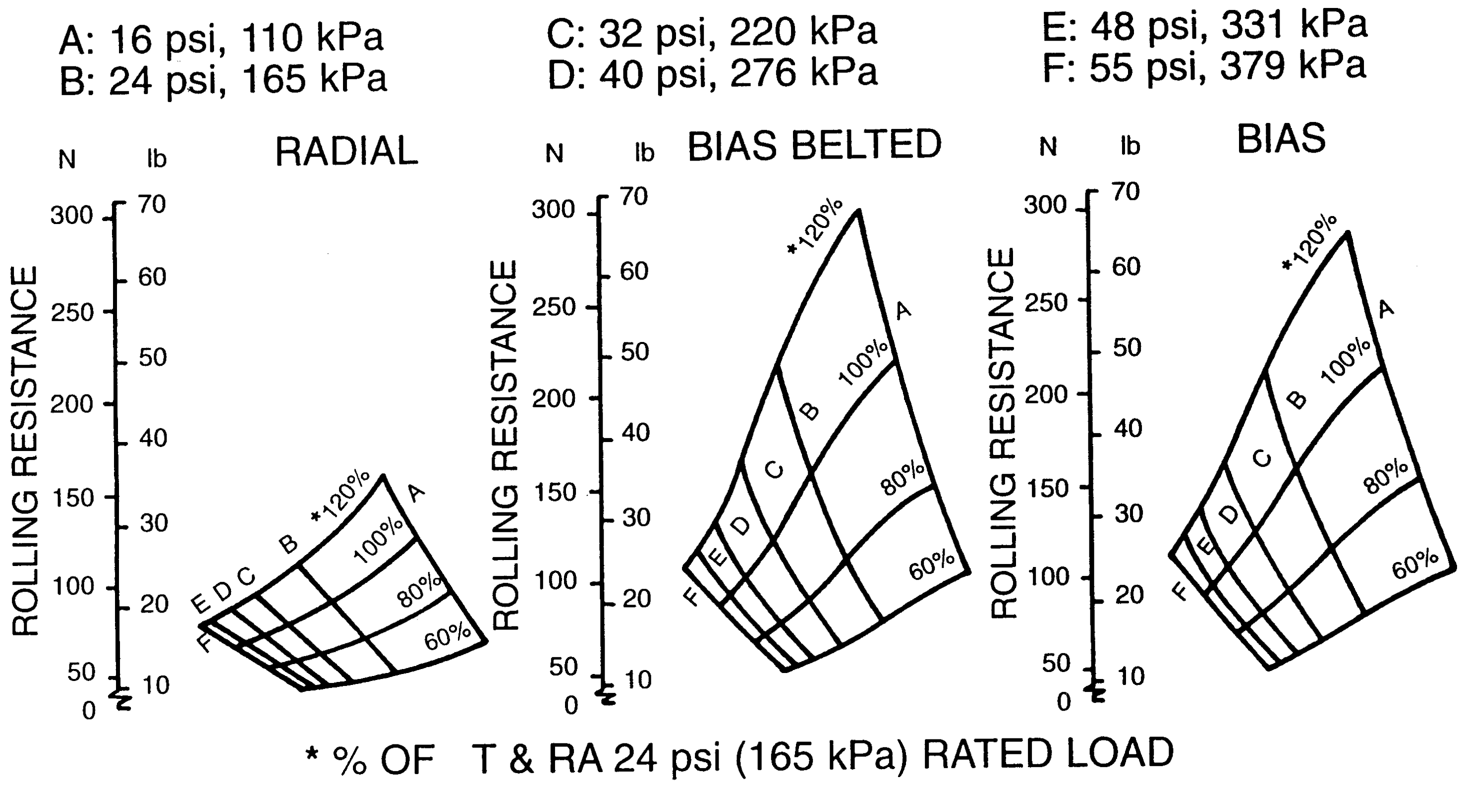


Fig. 1.7 Variation of rolling resistance of radial-ply, bias-belted, and bias-ply car tires with load and inflation pressure. (Reproduced with permission of the Society of Automotive Engineers from reference 1.11.)

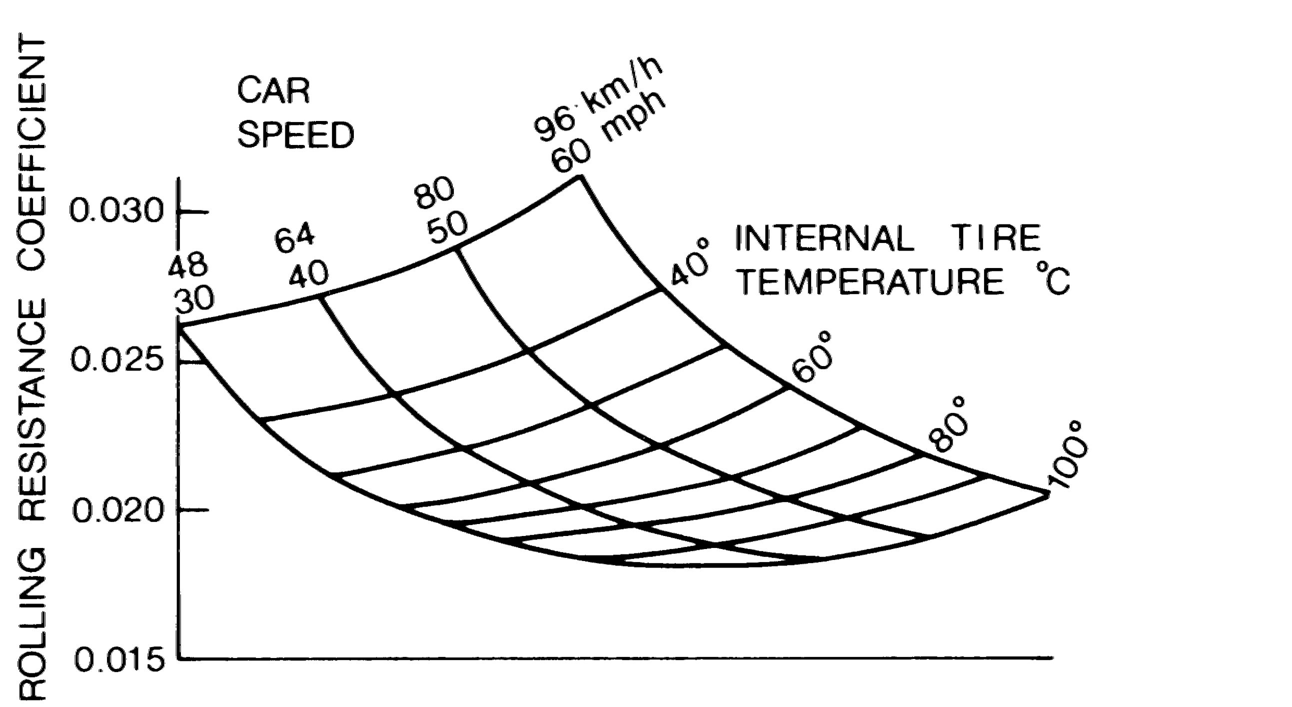
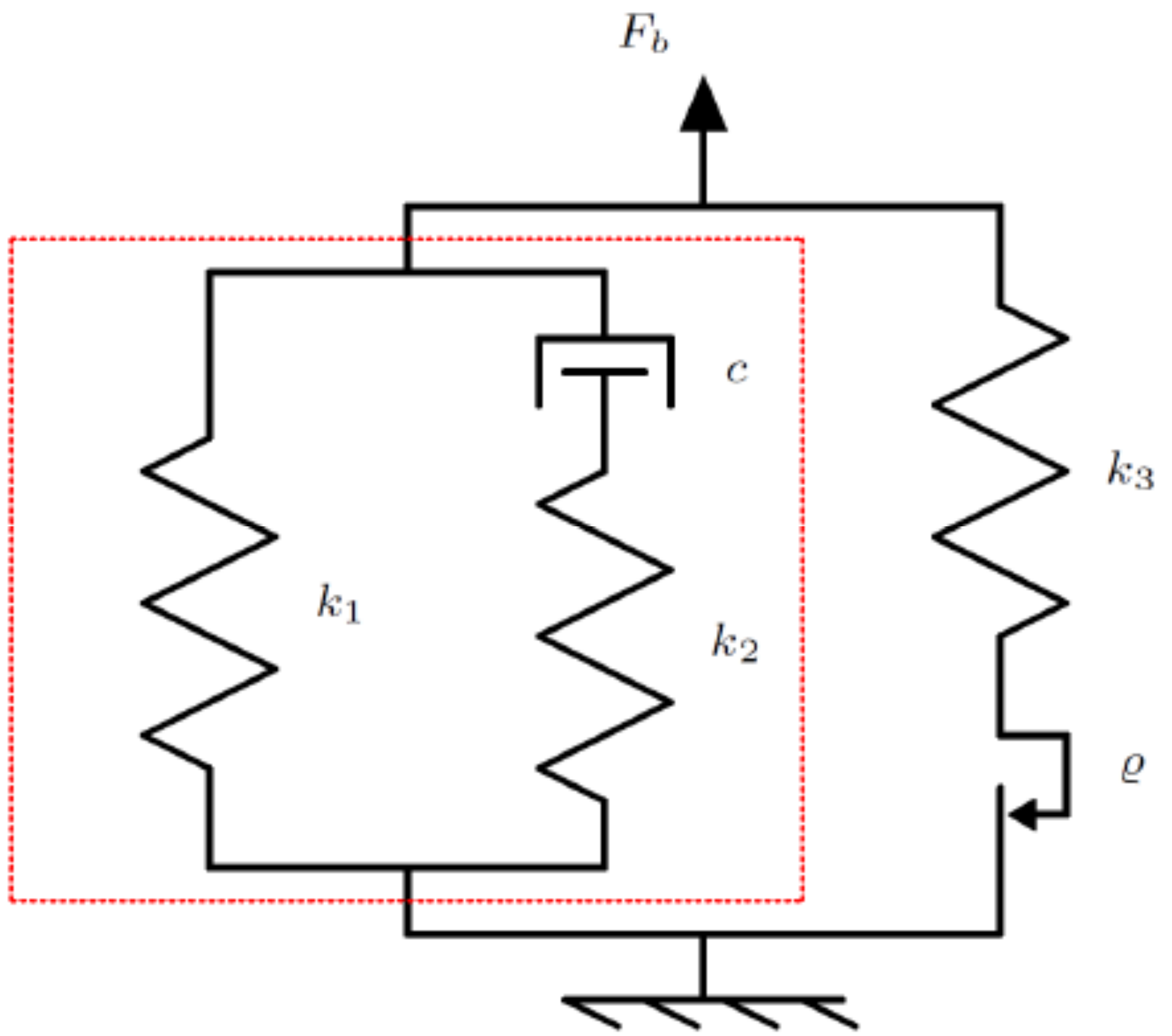


Fig. 1.11 Effect of internal temperature on rolling resistance coefficient of a car tire. (Reproduced with permission of the Council of the Institution of Mechanical Engineers from reference 1.5.)

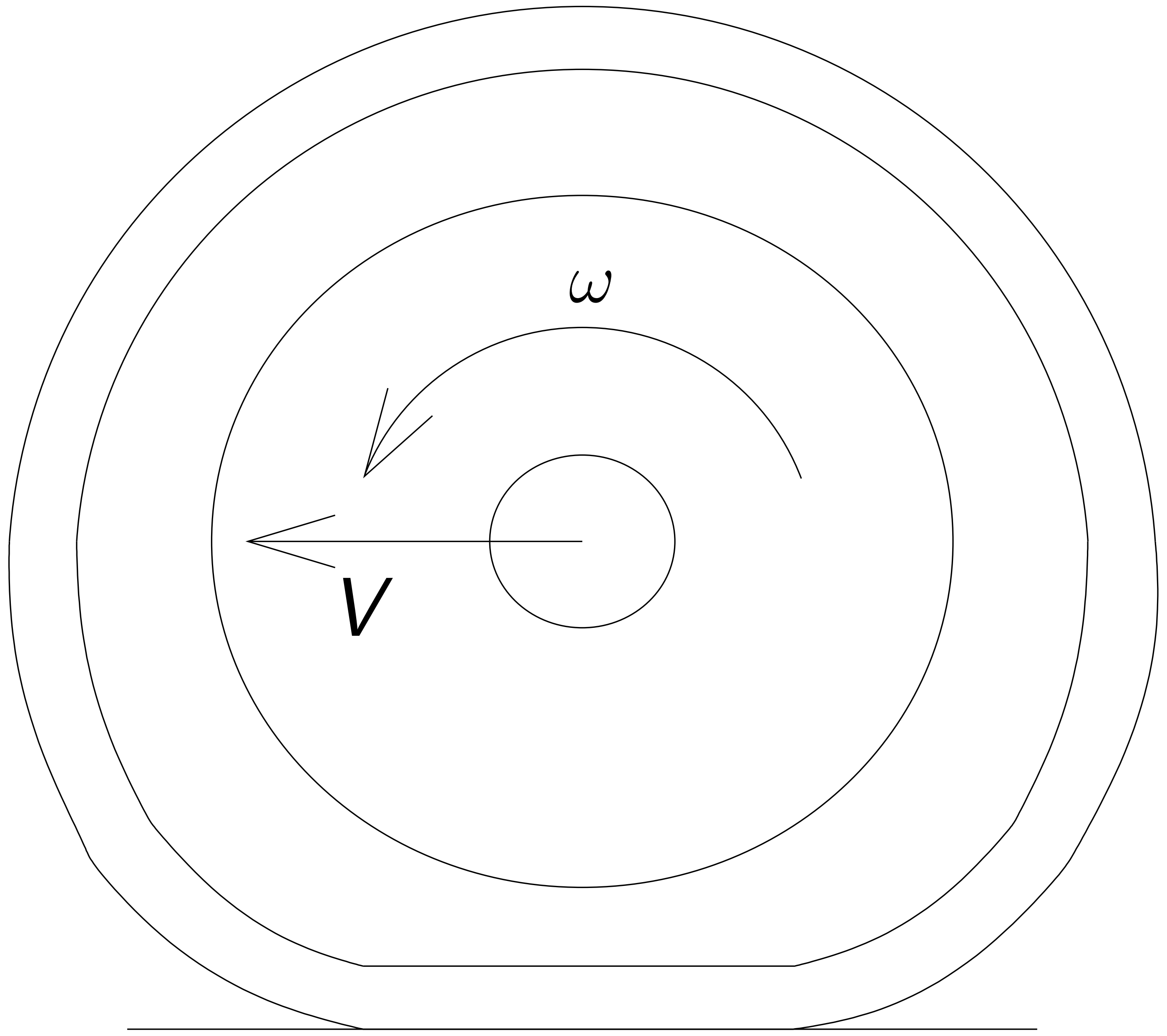
An Example of a Physical Model

Figure from the paper
Parametrisation of a rolling resistance model for extending the brush tyre model
Ydrefors, L., Åsenius, M., Jansson, H. Kharrazi, S., Hjort, M., Åslund, J.



Tyre Modeling: The Brush Model

A Tire Under the Action of a Driving Torque



Definitions:

- Rolling radius of a free-rolling tire: $r = V/\omega$,
- Effective rolling radius under the action of a driving torque: $r_e = V/\omega$,

  where V is the linear speed of the tire center, and ω is the angular speed.

A Tire Under the Action of a Driving Torque

Longitudinal slip

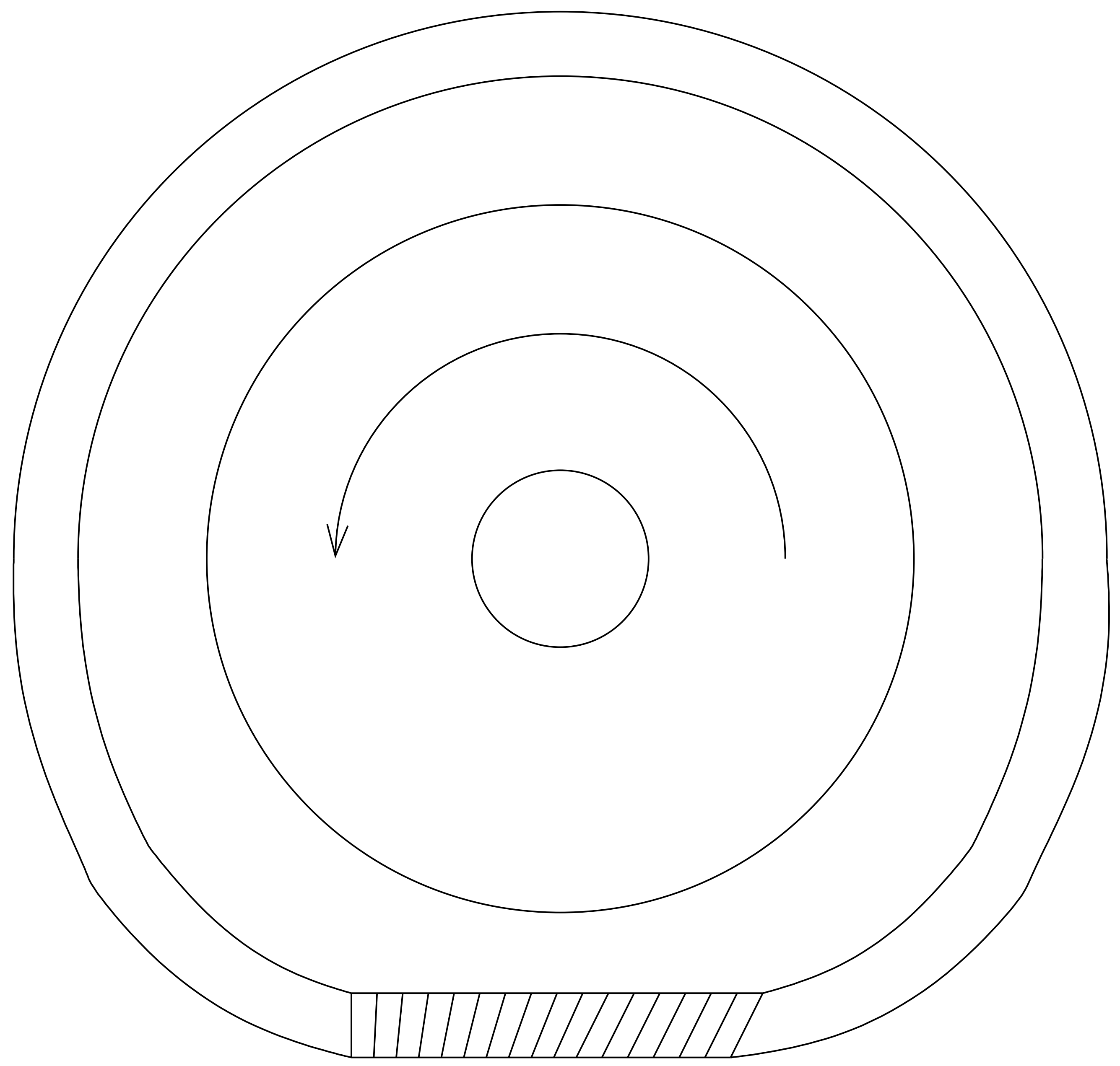
$$i = \left(1 - \frac{V}{\omega r}\right) = \left(1 - \frac{r_e}{r}\right)$$

Limit cases:

- Free-rolling tire: $i = 0$
- The tire is not moving: $i = 100\,\%$ om$V = 0$

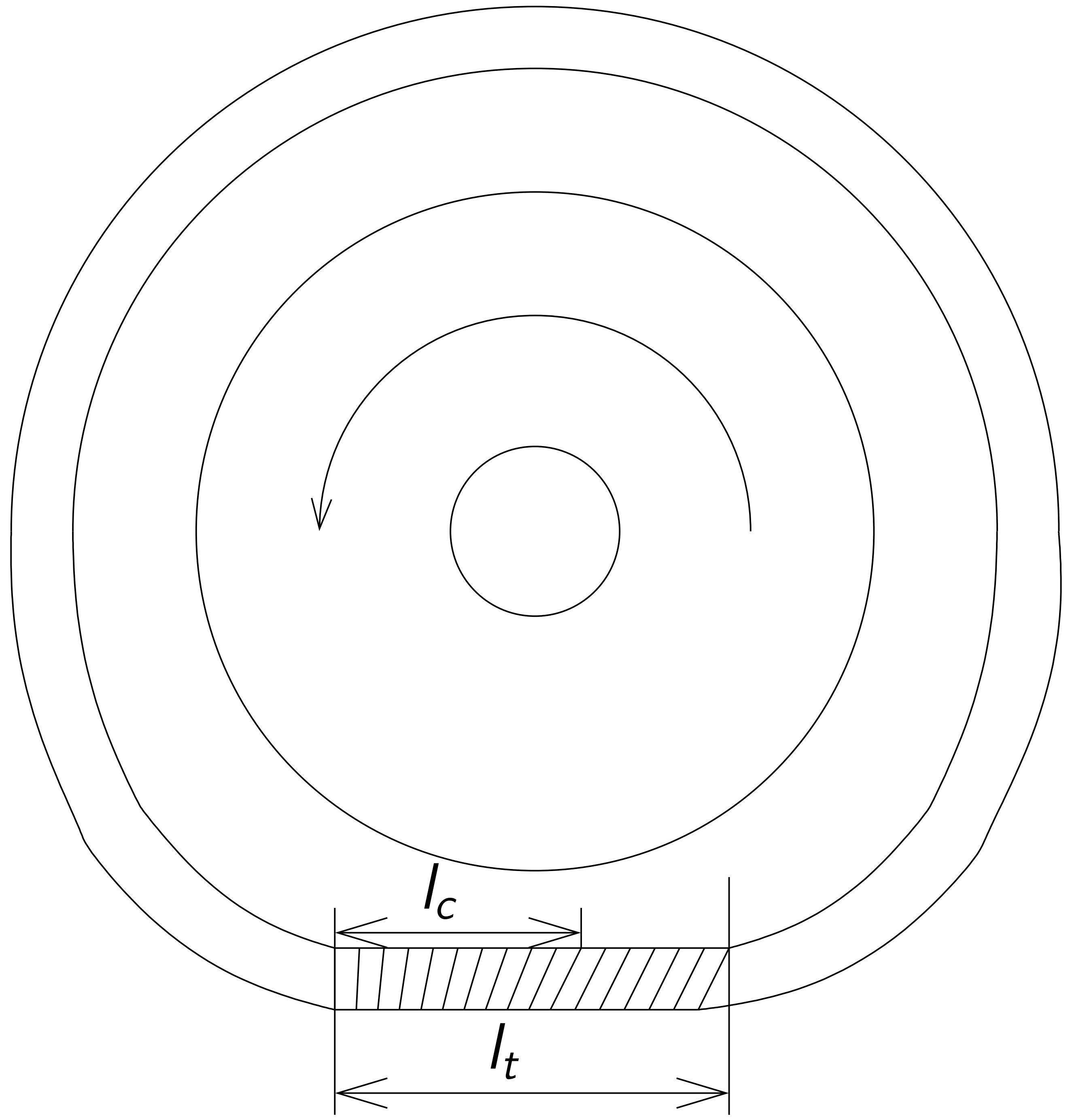
Driving Wheel: The Brush Model

The brush model is a simple physical model of tire. The tread of the tire is modeled as elastic bristles attached to the rim, and longitudinal force is generated by the deflection of the brush elements.



Driving Wheel: The Brush Model

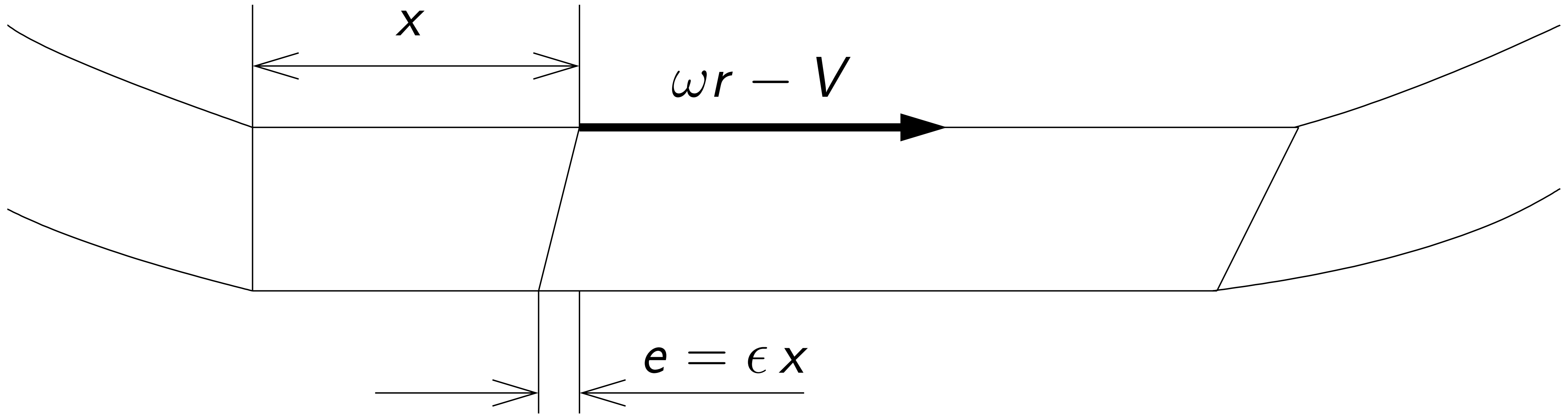
The contact patch is assumed to rectangular and can be divided into an **adhesion region** ($0 \leq x \leq l_c$), and a **sliding region** ($l_c \leq x \leq l_t$). *Vilozon* and *glidzon* in Swedish.



Driving Wheel: The Brush Model

Now, the objective is to find the length of the adhesion region l_c. When does the longitudinal force becomes so large that the bristles begins to slide?

Consider a bristle in the adhesion region. The velocity at the rim is $\omega r - V$.



The time since the bristle first touch the ground is $t = x/(\omega r)$.

The deflection at the distance x is:

$$e(x) = (\omega r - V)\frac{x}{\omega r} = \left(1 - \frac{V}{\omega r}\right)x = ix$$

Driving Wheel: The Brush Model

- Using a linear model for the relation between deflection and longitudinal force per unit of length:

$$\frac{dF_x}{dx} = k_t e = k_t i x$$

- It is assumed that normal force W is uniformly distributed in the contact region,

$$\frac{dF_z}{dx} = \frac{W}{l_t}$$

where l_t is the length of the contact region.

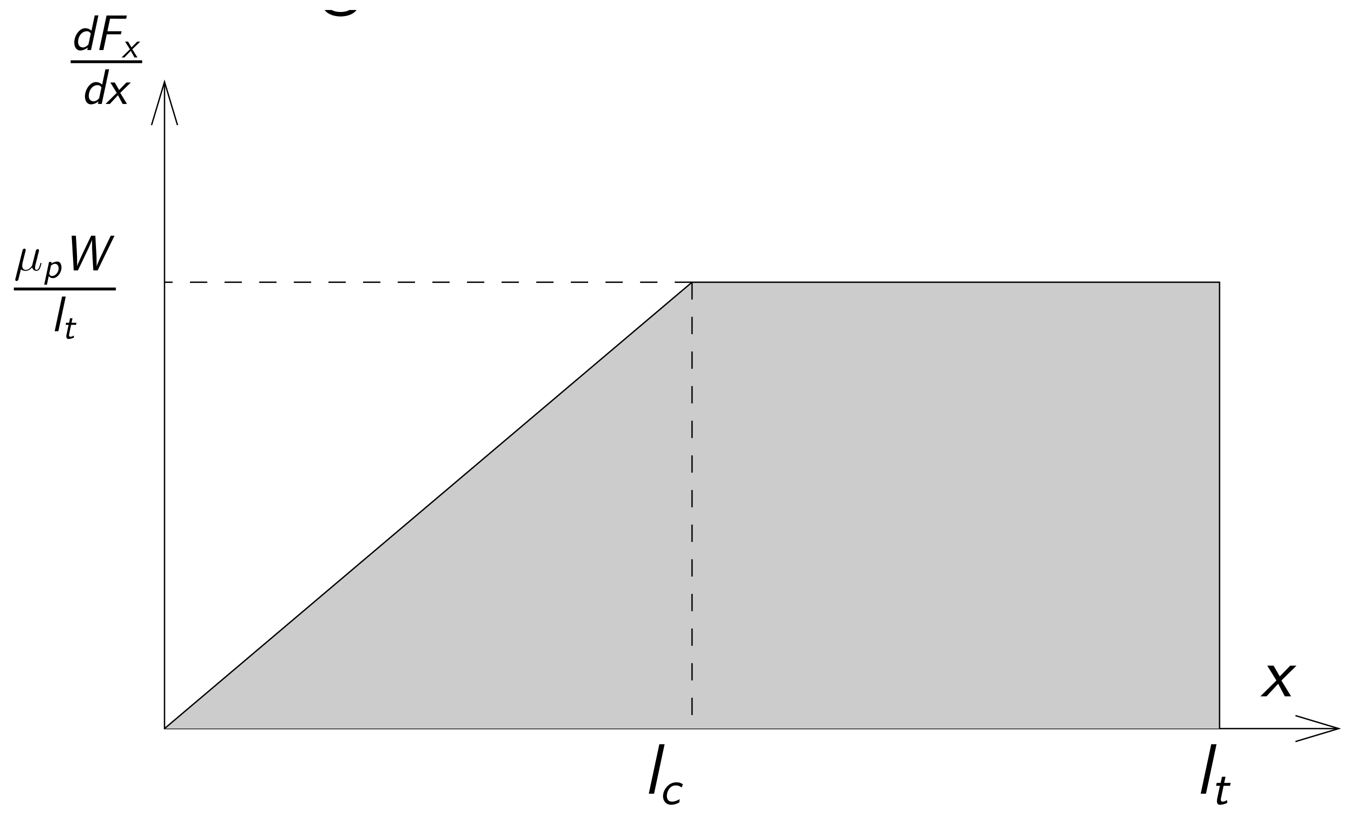
- Assumption: The bristle will not slide if

$$\frac{dF_x}{dx} < \mu_p \frac{dF_z}{dx}$$

where μ_p is the coefficient of friction.

Driving Wheel: The Brush Model

The sliding condition can be written



$$k_t i x < \mu_p \frac{W}{l_t}$$

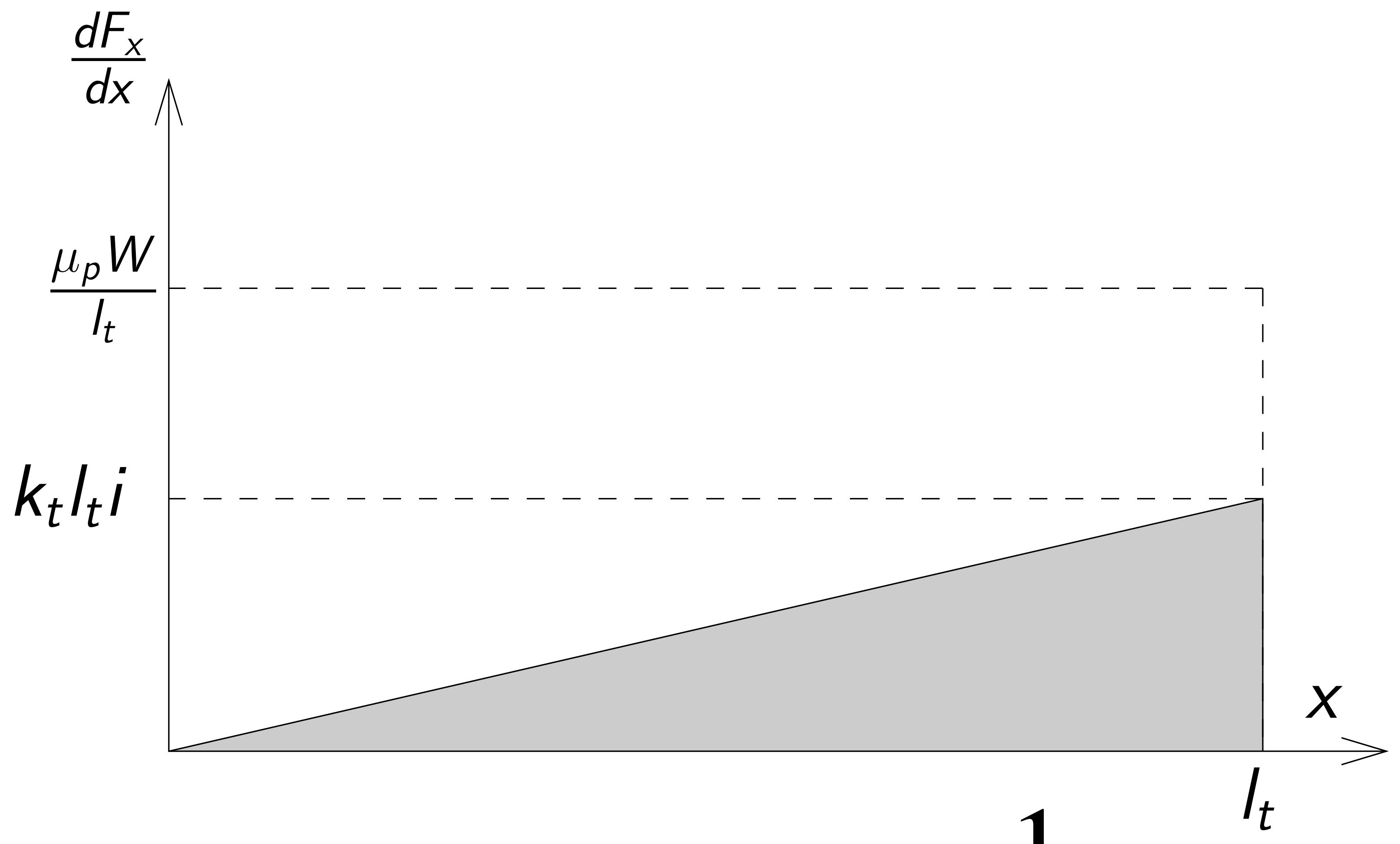
First case: When is there no sliding region?

Answer: When $x = l_t$ fulfills the condition above, i.e.

$$k_t l_t i < \frac{\mu_p W}{l_t} \quad \text{or} \quad i < \frac{\mu_p W}{k_t l_t^2} \equiv i_c$$

Driving Wheel: The Brush Model

The distribution of the longitudinal force in this case ($i < i_c$)



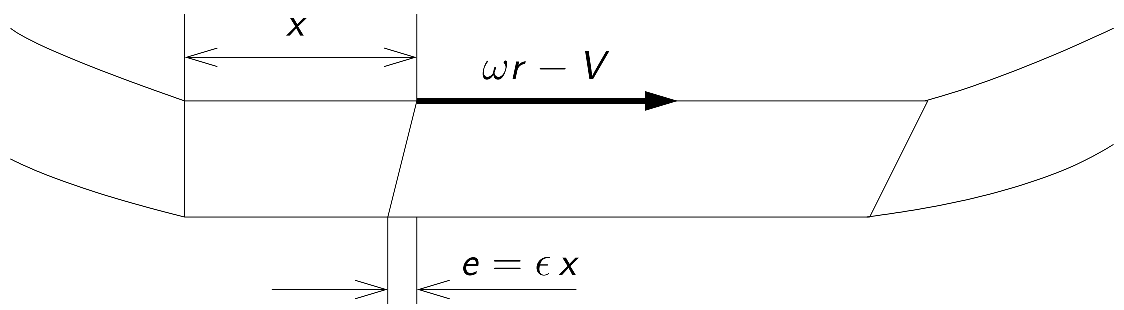
$$F_x = \text{Area of the shaded region} = \frac{1}{2} k_t l_t^2 i \equiv C_i i$$

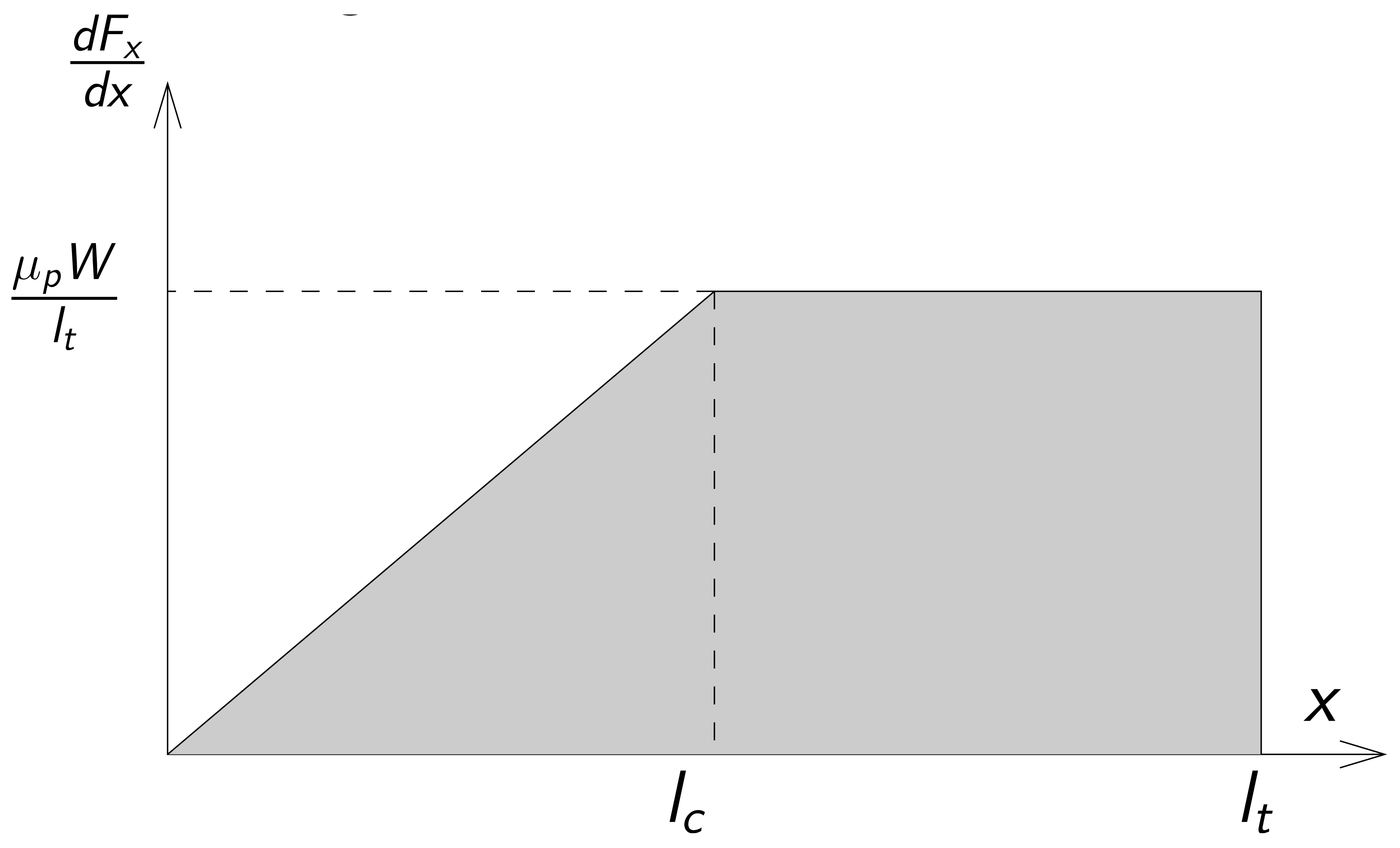
In the limit case $i = i_c = \dfrac{\mu_p W}{k_t l_t^2}$ is $F_x = \dfrac{1}{2} k_t l_t^2 \dfrac{\mu_p W}{k_t l_t^2} = \dfrac{\mu_p W}{2} \equiv F_{xc}$

Driving Wheel: The Brush Model

The second case: There is a sliding region ($i > i_c$).

The distribution of the longitudinal force in this case:



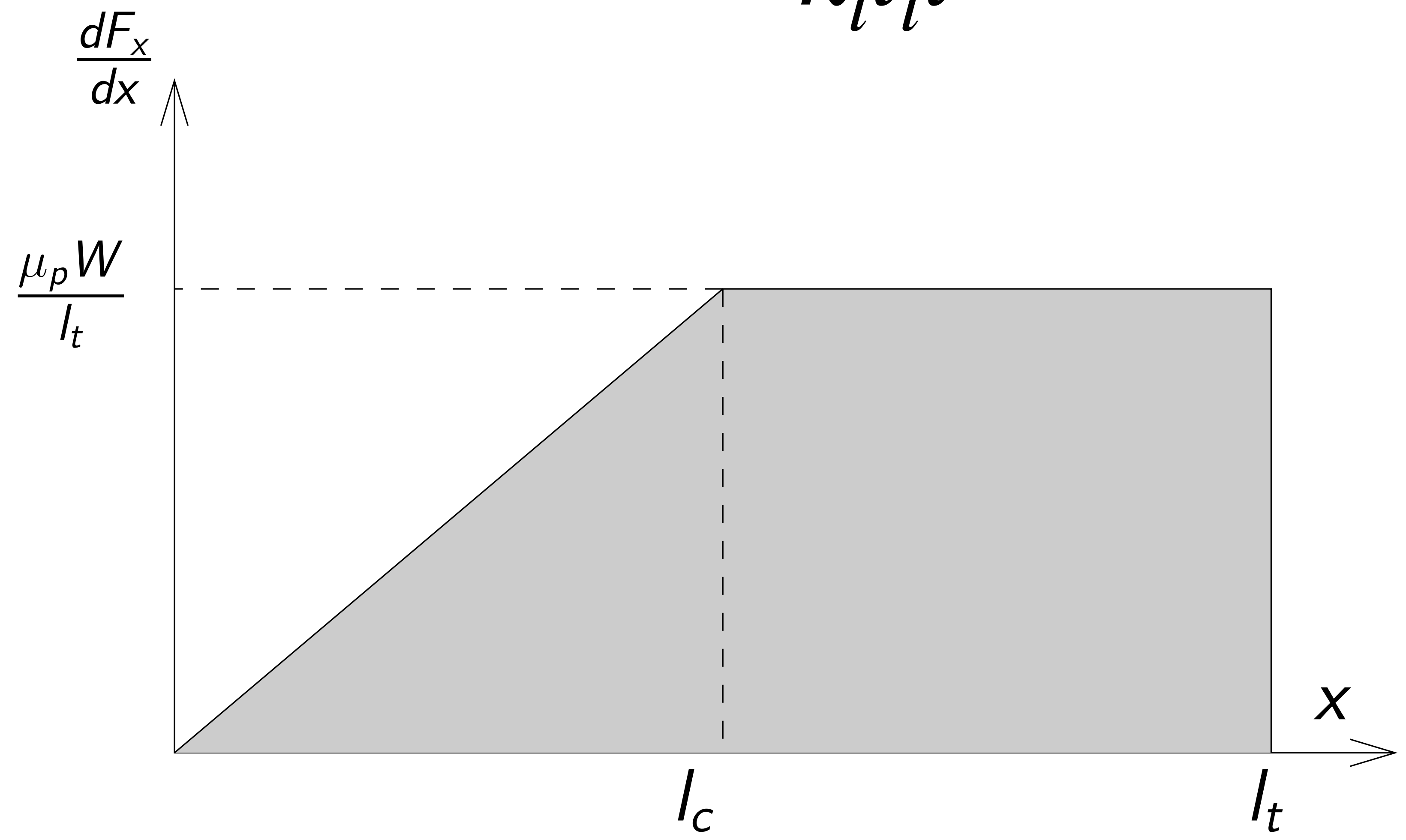


The objective is to calculate F_x, and the first step is to calculate the length of the adhesion region l_c.

Driving Wheel: The Brush Model

Solution: Recall that the bristle will not slide if $k_t i x < \mu_p W / l_t$, i.e.,

$$x \leq \frac{\mu_p W}{k_t l_t i} \equiv l_c$$



The longitudinal force is equal to the shaded area

$$F_x = \frac{1}{2}\frac{\mu_p W}{l_t} l_c + \frac{\mu_p W}{l_t}(l_t - l_c) = \mu_p W\left(1 - \frac{1}{2}\frac{l_c}{l_t}\right)$$

The Brush Model: Summary

Critical values if longitudinal slip and force:

$$i_c = \frac{\mu_p W}{k_t l_t^2} = \frac{\mu_p W}{2C_i} \text{ och } F_{xc} = \frac{\mu_p W}{2} = C_i i_c$$

There is no sliding region when $i \leq i_c$ eller $F_x \leq F_{xc}$ and in this case

$$F_x = \frac{k_t l_t^2}{2} i = C_i i$$

If $i > i_c$ or $F_x > F_{xc}$, then the length of the adhesion region is

$$l_c = \frac{\mu_p W}{k_t l_t i}$$

and the longitudinal force is

$$F_x = \mu_p W \left(1 - \frac{1}{2} \frac{l_c}{l_t} \right) = \mu_p W \left(1 - \frac{\mu_p W}{4 C_i i} \right)$$

Braking Wheel: The Brush Model

The skid is defined

$$i_s = \left(1 - \frac{\omega r}{V}\right) = \left(1 - \frac{r}{r_e}\right)$$

when a braking torque is applied to the wheel.

Limit cases:

- Free-rolling tire: $i_s = 0$
- Locked wheel:$i_s = 100\,\%$

Relations between i and i_s:

$$i = -\frac{i_s}{1 - i_s} \text{ and } i_s = -\frac{i}{1 - i}$$

Braking Wheel: Summary

$$C_s = \left.\frac{\partial F_x}{\partial i_s}\right|_{i_s=0}$$

Critical values of skid and longitudinal force

$$i_{sc} = \frac{\mu_p W}{2C_s + \mu_p W} \text{ and } F_{xc} = \frac{C_s i_{sc}}{1 - i_{sc}} = \frac{\mu_p W}{2}$$

No slide region ($i_s < i_{sc}$):

$$F_x = \frac{C_s i_s}{1 - i_s}$$

With slide region ($i_s \geq i_{sc}$):

$$F_x = \mu_p W\left(1 - \frac{\mu_p W(1 - i_s)}{4C_s i_s}\right)$$