

TSFS02 — Vehicle Dynamics and Control

Computer Exercise 1:  
Brake-Force Distribution and Slip Control

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# 1 Introduction

The purpose of this exercise is to learn about the impact of brake-force distribution and anti-lock brakes, and how they affect the vehicle's longitudinal and lateral dynamics. To get a feel for this you will elaborate with the brake-force distribution, implement your own Electronic Brake-force Distribution (EBD) controller, and implement a slip controller, which is an alternative to the Anti-lock Braking System (ABS). You will show how these systems can be beneficial in terms of safety by running different simulation experiments and compare the results. The software tool you will use is Matlab, for which a code skeleton is provided and based on this you are supposed to implement your controllers.

## 1.1 Examination

To pass this exercise you should have fulfilled the following:

- Solved the preparation tasks.
- Solved all the computer exercise tasks.
- Answered all questions, with motivated and thoughtful answers.

The examination is done by presenting your results and answers to a course assistant at the scheduled exercise session. To speed up the examination process, it might be a good idea to present the tasks as you complete them, instead of saving them all to the end.

## 2 Preparation Tasks

The tasks in this section (i.e., Tasks 1–5) are preparation tasks that should be solved before starting with the computer exercises. Please verify with an assistant that you have solved these tasks correctly before starting with proceeding (since your code implementations will be based on the equations you derive in the preparation tasks).

### **Task 1 – Brake-Force Distribution: *Simultaneous wheel lock-up***

Calculate the brake-force distribution so that both front and rear wheels lock up at the same time. Use the parameters in Table 1 in the appendix. For simplicity, neglect air and rolling resistance, and assume the road inclination is zero.

### **Task 2 – Brake-Force Distribution: *EBD control algorithm***

The brake-force distribution you calculated in the previous task is designed for maximum braking, and for a surface with the specified friction coefficient. This can be circumvented with an Electronic Brake-force Distribution (EBD) that varies the front-to-rear brake-force distribution according to the deceleration. Derive an expression for the brake-force distribution, so that all wheels lock at the same time, **conditioned on the fact that the friction coefficient is unknown**. You are free to use the available measurement signals shown in Table 2 in the appendix.

### **Task 3 – Slip Control: *Wheel dynamics***

Consider the dynamics of a wheel, the rotational acceleration  $\dot{\omega}$  is a function of the tire–road force  $F_b$  and by the applied braking torque  $T$ . Write down the corresponding equation of motion.

### **Task 4 – Slip Control: *Control algorithm***

In this task you are supposed to write down the basic control sequences of your slip controller, which you will use as a basis for the implementation. Synthesize your controller using the first order dynamics

$$\dot{\omega} - \dot{\omega}_{des} = k(\omega - \omega_{des}) \quad (1)$$

where  $\omega_{des}$  is the desired rotational wheel velocity and  $k$  is the controller gain.

- Replace  $\dot{\omega}$  with the wheel dynamics from Task 3 and write down the resulting controller using the applied braking torque as output.
- Formulate  $\omega_{des}$  as a function of the desired slip ratio  $i_{des}$ . (The slip ratio is defined in (2).)
- Calculate  $\dot{\omega}_{des}$ .
- Define a slip reference value  $i_{des}$  for your controller. (Hint: You can use Figure 1 for this.)
- Formulate a way to estimate the feedforward braking force  $F_b$ . Use either a fixed value or the sensors. Note that  $F_b$  in this case is the braking force for a single wheel, not all four.
- Which measurement signals do you need? Compare with Table 2 to ensure that your needed measurement signals are available.

### Task 5 – Slip Control: *Control algorithm*

Think about the function of the feedback gain  $k$  in Task 4, should it be positive or negative?

## 3 Computer Exercise Tasks

Before starting with the computer exercises, download the code skeleton from the course homepage. In the folder open MATLAB. <sup>1</sup> In the appendix you will find data for some vehicle parameters, tire force behavior, and available sensors, that can be useful when solving the tasks.

### 3.1 Brake-Force Distribution

Electronic brake-force distribution distributes the braking force between the wheels according to the overall weight distribution of the vehicle.

### Task 6 – Brake-force distribution: Wheel lock-up

In this task you are going to compare different levels of brake-force distributions using MATLAB. Open the file named Task\_6\_WheelLockup.m. In this experiment the brake-force is applied by increasing it from zero, while driving in a straight line.

- Adjust the brake balance to front only, by setting the parameter  $Kbf = 1$ . Run the simulation, look at the results, and save.
- Adjust the brake balance to rear only (set  $Kbf = 0$ ). Run the simulation, look at the results, and save.
- Set the brake balance to the value you calculated in Task 1. Run the simulation, study the results, and save.

Find the peak deceleration reached during braking and fill it in below.

Front wheels only ( $K_{b,f} = 1$ ): \_\_\_\_\_

Rear wheels only ( $K_{b,f} = 0$ ): \_\_\_\_\_

Calculated brake-force distribution ( $K_{b,f}$  from Task 1): \_\_\_\_\_

<sup>1</sup>In this lab, you will have to use a program called CasADi. This program is preinstalled in all Linux-based computer labs by accessing the folder 'courses/tsfs12/casadi/'. You add the installed directory with: `addpath /courses/tsfs02/casadi`. If you are using your own computer, download and install CasADi according to the instructions on <https://web.casadi.org>

## Task 7 – EBD implementation

Implement an EBD in Matlab, based on the expression you derived in Task 2. The EBD is implemented in the function `MyEBD.m`. The input of this function contains the acceleration `ax` and parameters struct `p`. Double click `p.max` in the folder and then double click `p` in the Matlab Workspace then you will find all available parameters. Based on the input of this function and the necessary parameters, write the code to calculate `kbf`.

## Task 8 – EBD evaluation

In this task you will evaluate your EBD controller with three different experiments. The first is the “ramp braking” from Task 6. The second is similar to “ramp braking”, but for a lower friction coefficient (scaled down to  $\mu \approx 0.3$ ). The third experiment is a “braking while cornering” test, where the brakes are applied while turning.

- a) **Straight-line braking:** Open the file `Task_8_StraightlineBraking.m`. Run the simulation and analyze the results.

Do the front and rear wheels lock up roughly at the same time?

How does the brake-force distribution change with increased braking effort?

- b) **Low-friction braking:** Open the file `Task_8_LowfrictionBraking.m`.

*i)* Set the brake-force distribution to a “front-heavy setup” (e.g.,  $K_{b,f} = 1$ ). This is achieved by choosing the FH as the EBD mode in the file, and run the simulation.

*ii)* Set the brake-force distribution to a “rear-heavy setup” (e.g.,  $K_{b,f} = 0$ ). This is achieved by choosing the RH EBD mode in the file, and run the simulation.

*iii)* Run the experiment with your EBD controller activated. This is achieved by choosing the ON as the EBD mode in the file, and run the simulation.

How does the wheel lock-up differ between these modes?

How does the braking performance change with respect to these modes?

- c) **Braking while cornering:** Open the file `Task_8_BrakingCornering.m`.

*i)* Set the brake-force distribution to a “front-heavy setup” (e.g.,  $K_{b,f} = 1$ ). This is achieved by choosing the FH as the EBD mode in the file, and run the simulation.

*ii)* Set the brake-force distribution to a “rear-heavy setup” (e.g.,  $K_{b,f} = 0$ ). This is achieved by choosing the RH as the EBD mode in the file, and run the simulation.

*iii)* Run the experiment with your EBD controller activated. This is achieved by choosing the ON as the EBD mode in the file, and run the simulation.

How does the wheel lock-up differ between these modes?

How does the braking performance change with respect to these modes?

Why is the lateral dynamics affected in this way? (Hint: See Figure 2.)

## Task 9 – Benefits of EBD

Name two benefits with EBD, compared to a fixed brake-force distribution, that you have learned from the above tasks:

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## 3.2 Slip Control

Slip control is an alternative to ABS systems. ABS systems operate by reducing the braking pressure when the slip ratio approaches the unstable region, where if the braking force is kept constant the wheel will eventually lock (See Figure 1). In contrast, slip control systems control the slip ratio towards a desired value.

### Task 10 – Slip control implementation

Implement your controller in the function `MySC.m`. You are supposed to use the input of the function as well as the necessary parameters Synthesized in the struct `p` to compute the wheel torques you derived in Task 4.

### Task 11 – Slip control evaluation

You will now evaluate your controller.

- a) **Straight-line full braking:** Open the file `Task_11_StraightlineFullBraking.m`.
  - i) Make sure that the SC mode is OFF and run the experiment. Note that the wheels should lock up even when the SC controller is deactivated in this case.
  - ii) Run the experiment with your SC mode is ON.

Show that your slip controller acts as intended.

Estimate the braking distance (from when the brakes are applied):

Locked wheels: \_\_\_\_\_

With SC active: \_\_\_\_\_

- b) **Braking while cornering (full braking):** Open the file `Task_11_BrakingWhileCornering.m`.
  - i) Run the experiment when the SC mode is OFF.
  - ii) Run the experiment when the SC mode is ON.

How is the steering ability affected when the brakes are applied, with and without slip control?

### Task 12 – Benefits of slip control

Name the benefits of controlling the slip ratio that you have learned from these exercises:

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## A Vehicle and Tire Data

Table 1 specifies a few vehicle parameters, and in Table 2 the different measurement signals, available for the EBD and slip controller implementations, are listed.

Figure 1 show how the longitudinal tire force depends on the slip ratio  $i$ , with the slip ratio defined as

$$i = \frac{R_w \omega - v_x}{v_x} \quad (2)$$

where  $R_w$  is the wheel radius,  $\omega$  the wheel rotational velocity, and  $v_x$  the longitudinal velocity. Note that this slip definition returns a negative slip,  $i < 0$ , for braking. In Figure 2 the lateral force is shown, as a function of slip angle, for different slip ratios (i.e. for different levels of braking effort).

*Table 1 Vehicle parameters.*

Parameter	Description	Value	Unit
$m$	Total vehicle mass	2100	kg
$h$	CoG height	0.50	m
$L$	Wheelbase	2.8	m
$l_1$	CoG to front axle	1.3	m
$R_w$	Wheel radius	0.3	m
$J_w$	Moment of inertia of each wheel	2.0	kgm <sup>2</sup>
$\mu_x$	Friction coefficient	1.1993	

*Table 2 Vehicle sensors for the EDB and slip controllers.*

Variable	Description	Unit
$a_x$	Longitudinal acceleration	m/s <sup>2</sup>
$v_x$	Longitudinal velocity	m/s
$T_{fl}$	Wheel torque; front left	Nm
$T_{fr}$	Wheel torque; front right	Nm
$T_{rl}$	Wheel torque; rear left	Nm
$T_{rr}$	Wheel torque; rear right	Nm
$\omega_{fl}$	Wheel velocity; front left	rad/s
$\omega_{fr}$	Wheel velocity; front right	rad/s
$\omega_{rl}$	Wheel velocity; rear left	rad/s
$\omega_{rr}$	Wheel velocity; rear right	rad/s

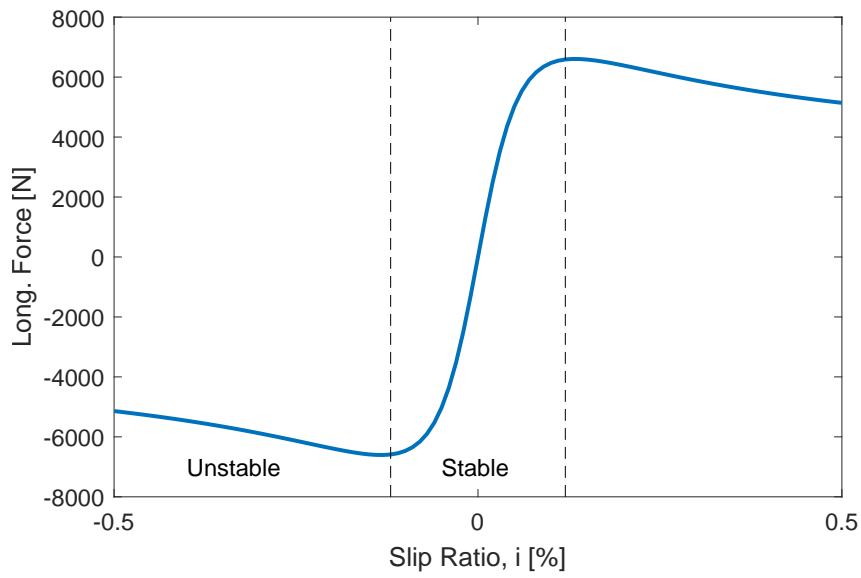


Figure 1 Longitudinal tire force.

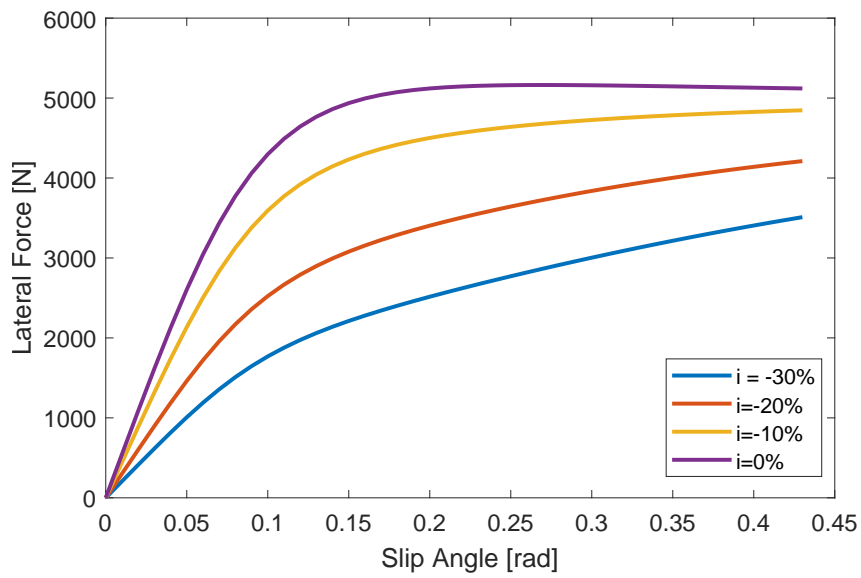


Figure 2 Lateral tire force.